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# ANALYSIS OF THE PION-NUCLEUS ELASTIC SCATTERING USING LOCAL KISSLINGER POTENTIAL

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**In the present study we investigate pion-nucleus elastic scattering from $^{16}$O and $^{40}$Ca in the pion kinetic energy range of 114 to 292MeV. A FORTRAN program is constructed to calculate the local Kisslinger potential for pion-nucleus interaction for first- and second- order interactions. The constructed potential takes into account all considered corrections made by others such as Coulomb correction, Perey factor and Ericson-Ericson Lorentez-Lorentez effect. The sensitivity of cross section to different forms of target densities are studied. Also, the effect of neglecting second-order parameters is studied. Good reproductions of the data are obtained by using both first- and second order parameters. The cross section is produced well with all densities.**

**Keywords:** Pion-nucleus elastic scattering; Optical model; Local Kisslinger potential;

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## I. INTRODUCTION

The scattering of pions by nucleus has played a particularly significant role in the development of nuclear physics. On the one hand, it is readily accessible to experimental investigations because of the availability of high intensity pion beams over a wide range of energies. Because of its strong interaction with the nucleus, the pion can be used to probe nuclear density and energy levels. It is interesting to note that the results obtained by the study of pion-nucleus scattering could be used to make

comparisons with information obtained from nucleon-nucleus scattering[1]. The pion-nucleus scattering has been studied experimentally[2-4] for different target nuclei and different range of energies. The most interesting and rich area is the resonance area of a middle energy.

To analyze the experimental data, the construction of nuclear models has been approached. Many theoretical approaches are found in literature[5-17]. One of these models is the optical model which assumes that the interaction between incident particle and nucleus can be described by a complex potential: the real part describes the scattering of the incident particle, and the imaginary part represents their absorption. In this model, the nucleus is considered as a system of particles directly independent of each other and moving in a certain average nuclear complex potential. The pion-nucleus optical potential was first proposed by Kisslinger in Ref.[5] as a nonlocal potential based on multipole scattering theory. Later on, it was transformed to the more simple local form in Refs.[6,7]. Johnson and Satchler modified the local kisslinger potentials to include the so called Perey factor[14]. Based on this optical model the large body of experimental data was fitted in Refs.[14-17] at the pion energies from 20 to 291 MeV.

The Kisslinger local potential form is constructed by summing both s-wave and *p*-wave pion-nucleon interactions and affected by the values of the pion-nucleon scattering amplitude parameters namely the density distribution model, the value of Ericson_Ericson Lorentz_Lorenz polarization parameter and Perey factor. The pion-nucleon scattering amplitude depends on complex first-order and second-order interaction parameters which are related to the free pion-nucleon scattering through the phase shifts. Previous work claimed that the second-order parameters are required only at lower energies $T_\pi \leq 80$ MeV, but these parameters make no differences in the calculations at higher energies, so they were set to be zero[15,16,18]. While in other studies[12,14,17]the second-order parameters were taken into account.

In this paper we follow Ref.[14] to derive the Kisslinger local potential from first- plus second-order microscopic theory at both low and resonance energies. A FORTRAN code for this potential is constructed. The pion elastic scattering differential cross sections at intermediate energy are investigated by inserting our calculated potential into FRESCO code[19]. We present the study of sensitivity of the differential cross sections to the choice of only first-order or first- and second order interaction parameters. Also, the sensitivity of the differential cross sections to the different density distribution forms of target nuclei is studied.

## II. FORMALISM

Since the Kisslinger potential satisfies all features of pion-nucleus interaction, it was suggested that the Kisslinger potential is more suitable for describing the pion-nucleus elastic and inelastic scattering[14-16]. Thus, the Kisslinger potential form[5] is the most common way to describe the pion-nucleus interaction. It contains explicitly terms which originate in the *p*-wave pion-nucleon interaction. The nonlocal Kisslinger potential is given by:

$$U_k(r) = \frac{(\hbar c)^2}{2\omega}\{q(r) + \boldsymbol{\nabla}\cdot\alpha(r)\boldsymbol{\nabla}\},$$

where $\omega$ is the total energy of the pion in the center of mass system, $q(r)$ primarily results from the *s*-wave part and $\alpha(r)$ results from the *p*-wave part of the pion-nucleon interaction, we will discuss the explicit form of $q(r)$ and $\alpha(r)$ later.

Johnson and Satchler[14] used the Krell-Ericson transformation, which leads from the Klein-Gordon equation for pion scattering to a local potential for the transformed wave function. The transformed wave function ψ satisfies a Schrödinger equation:

$$\left\{-\left(\frac{\hbar^2}{2\mu}\right)\nabla^2 + U_L + V_c\right\}\psi = E_{cm}\psi,$$

where $V_c$ is Coulomb potential:

$$V_c(r) = \begin{cases} \frac{Z_p Z_T e^2}{2R_c}\left(3 - \frac{r^2}{{R_c}^2}\right) & for \;\; r < R_c \\ \frac{Z_p Z_T e^2}{r} & for \;\; r \geq R_c \end{cases},$$

where $Z_p e$ and $Z_T e$ are the charges of the projectile and the target, respectively, and $R_c = r_c A^{1/3}, r_c = 1.2\, fm.$ $U_L$is nuclear local transformed potential $U_{\mathrm{L}}$ is given by:

$$U_L(r) = U_N(r) + \Delta U_C(r)\,,$$

where $U_N(r)$ is the nuclear potential:

$$U_N(r) = \frac{(\hbar c)^2}{2\omega}\left\{\frac{q(r)}{1-\alpha(r)} - \frac{k^2\alpha(r)}{1-\alpha(r)} - \left[\frac{\frac{1}{2}\nabla^2\alpha(r)}{1-\alpha(r)} + \left(\frac{\frac{1}{2}\nabla\alpha(r)}{1-\alpha(r)}\right)^2\right]\right\},$$

where $k$ is the wave number and $\Delta U_C(r)$ is the Coulomb correction term:

$$\Delta U_C = \frac{\alpha(r)V_C - \left(V_C^2/2\omega\right)}{1-\alpha(r)}\,,$$

and a reduced mass $\mu$ is:

$$\mu = \frac{M_\pi m_T}{(M_\pi + m_T)}\,,$$

with $m_T$ is the target mass and the effective pion mass is $M_\pi = \gamma_\pi m_\pi$, $\gamma_\pi$ is defined as:

$$\gamma_\pi = \frac{\chi + \gamma_l}{(1 + \chi^2 + 2\chi\gamma_l)^{\frac{1}{2}}}\,,$$

$$\chi = {m_\pi}/{m_T}\,,$$

$$\gamma_l = 1 + (T_\pi/m_\pi c^2)\,,$$

where $T_\pi$ is the pion bombarding energy in the laboratory system. The produced kinematic parameters are shown in Table I.

**TABLE I.** Produced kinematic parameters for use in a nonrelativistic Schrödinger equation that used in the present work with pion bombarding energy $T_\pi$ and effective energy $E_L$.

| Target | $T_\pi$(MeV) | $E_L$(MeV) | $M_\pi(u)$ | $\omega$(MeV) | $k(fm^{-1})$ |
|---|---|---|---|---|---|
| $^{16}O$ | 114 | 89.68 | 0.269577 | 246.5731 | 1.055315 |
| | 162 | 120.97 | 0.319886 | 291.2056 | 1.328463 |
| | 240 | 167.76 | 0.399162 | 362.5841 | 1.745313 |
| $^{40}Ca$ | 116 | 90.125 | 0.27315 | 252.7115 | 1.077915 |
| | 180 | 130.227 | 0.340828 | 314.796 | 1.444939 |
| | 290 | 195.518 | 0.459243 | 422.4472 | 2.046529 |

Both $q(r)$ and $\alpha(r)$ are complex and energy dependent. The explicit forms of $q(r)$ and $\alpha(r)$ are expressed in terms of the nuclear densities of the target nucleus. The predominantly *s*-wave interaction term $q$ may be written:

$$q(r) = q_0(r) + \Delta q(r) ,$$

where $q_0$ is purely *s*-wave, and $\Delta q$ arises only from the *p*-wave. Then the *s*-wave term is given by:

$$q_0(r) = -4\pi p_1(b_0\rho \mp b_1\Delta\rho) - 4\pi p_2\left(B_0\rho_{np} \mp B_1\rho\,\Delta\rho\right) ,$$

The *p*-wave part includes the Ericson_Ericson Lorentz_Lorenz (EELL) correction for the term linear in the density,

$$\alpha(r) = \frac{\alpha_1(r)}{1+\frac{1}{3}\zeta\alpha_1(r)} + \alpha_2(r) ,$$

where

$$\alpha_1(r) = 4\pi(c_0\rho \mp c_1\Delta\rho)/p_1 ,$$

$$\alpha_2(r) = 4\pi\left(C_0\rho_{np} \mp C_1\rho\,\Delta\rho\right)/p_2 .$$

The quantities $b_i$ , $c_i$ , $B_i$, and $C_i$ $(i = 0, 1)$ in equations (12), (14) and (15) are the complex and energy-dependent first- and second- order amplitude interaction parameters, respectively. The upper and lower sign refers to the $+$ $/-$ charge state of the pion. Also, $p_1 = (1 + \varepsilon)$ and $p_2 = (1 + \varepsilon/2)$ are kinematic transformation factors, with $\varepsilon = \omega/m_\pi c^2$ and $m_\pi$ is the mass of a nucleon. The angle transformation(*p*-wave) part of $q(r)$ is:

$$\Delta q(r) = -\tfrac{1}{2}\varepsilon\nabla^2\left[\alpha_1(r) + \tfrac{1}{2}\alpha_2(r)\right].$$

The $\alpha(r)\ and\ q(r)$ terms are calculated by the given equations are density dependence. We use $\rho(r) = \rho_n(r) + \rho_p(r)$ for the total density, $\Delta\rho(r) = \rho_n(r) + \rho_p(r)$ for the difference and $\rho_{np}(r) = 4\rho_n(r)\,\rho_p(r)$. We use simple two-parameter fermi, three-parameter fermi, and harmonic oscillator shapes for the density distribution for the studied nuclei. So that, two-parameter Fermi (2pF) is defined as[3,14]:

$$\rho_i(r) = \frac{\rho_{0i}}{\left[1+exp\left(\frac{r-c_i}{a_i}\right)\right]},$$

three-parameter Fermi (3pF) is defined as[3,20]:

$$\rho_i(r) = \frac{\rho_{0i}\left(1+\omega_i r^2/c_i^2\right)}{\left(1+exp\left(\frac{r-c_i}{a_i}\right)\right)},$$

and the harmonic oscillator (HO) is defined as[14]:

$$\rho_i(r) = \rho_{0i}\left[1 + \alpha_i\left(\frac{r}{c_i}\right)^2\right] exp\left[-\left(\frac{r}{c_i}\right)^2\right],$$

with $i = n$ for neutrons or $p$ for protons

It is of interest to decompose the expression (5) into its various terms:

$$U_N(r) = U_1(r) + U_2(r) + U_3(r),$$

where

$$U_1(r) = U_0(r) + \Delta U_1(r),$$

where $\Delta U_1$ is purely *p*-wave, while $U_0$ is proportional to the *s*-wave interaction but still influenced by the *p*-wave through the $1-\alpha(r)$ denominator

$$U_1(r) = \frac{(\hbar c)^2}{2\omega}\frac{q_0(r)}{1-\alpha(r)}\,,\ \Delta U_1(r) = \frac{(\hbar c)^2}{2\omega}\frac{\Delta q(r)}{1-\alpha(r)}\,,$$

The two terms $U_2$ and $U_3$ arise from the *p*-wave pion-nucleon interaction alone,

$$U_2(r) = -\frac{(\hbar c)^2}{2\omega}\frac{k^2\alpha(r)}{1-\alpha(r)}\,,$$

$$U_3(r) = -\frac{(\hbar c)^2}{2\omega}\frac{\frac{1}{2}\nabla^2\alpha(r)}{1-\alpha(r)} + \left(\frac{\frac{1}{2}\nabla\alpha(r)}{1-\alpha(r)}\right)^2.$$

By inserting the potentials $U_1$, $U_2$ and $U_3$ into Eq. (20), one can construct the local Kisslinger potential that can be used in the pion-nucleus scattering.

## III. PROCEDURE

The elastic scattering of $\pi^{\pm}$ from $^{16}$O and $^{40}$Ca in the pion kinetic energy range of 114 to 292 MeV are analyzed by using the local Kisslinger potential generated from Eq. (20). The local Kisslinger potential are generated by building a FORTRAN program by considering the following:

1) We construct our FORTRAN code to be applied at lower energies$T_\pi \leq$ 80MeV, $80 \leq T_\pi \leq 300$MeV, and at intermediate resonance energies.
2) We construct our FORTRAN code for three different forms of target densities, namely, 2pF, 3pF and HO densities.
3) Following the instruction of Ref. [14], we put $B_1 = C_1 = 0$ at the lower energies, $T_\pi \leq 80$ MeV, while $B_0 = B_1 = 0$ at intermediate energies.

4) At considered energies, we use the amplitude of first- and second-order parameters of Refs.[14,17] without adjusting any other free parameters in the optical potential.
5) For the sake of comparison, we use the amplitude first-order only produced in Ref.[18] to test the validity of the authors claim that second-order parameters are required only at lower energies $T_\pi \leq 80$ MeV.
6) The obtained potentials are fed into the computer code FRESCO[19] to compute the elastic scattering differential cross sections.

## IV. RESULTS AND DISCUSSION

The authors of Ref.[18] said that the second-order parameters are required only at lower energies $T_\pi \leq 80$ MeV, but these parameters make no differences in the calculations at higher energies. Thus, the local Kisslinger potential with modified first-order only with parameters from Ref.[18] is used to study the elastic scattering data of $\pi^{\pm}$ from $^{16}$O and $^{40}$Ca. Three different forms are used for target densities according to Eqs. (17), (18) and (19). For $^{16}$O, the density parameters are considered as 2pF[14], 3pF [20] and HO[14]. For $^{40}$Ca the density parameters are considered as 2pF and 3pF[3]. The $\rho_{0i}$ is calculated from normalization condition. Figure 1 shows the differential cross section of $\pi^+ + {}^{16}\mathrm{O}$ at 114, 162 and 240 MeV[4]. Figures 2 and 3 show the differential cross section of $\pi^+ + {}^{40}\mathrm{Ca}$ and $\pi^- + {}^{40}\mathrm{Ca}$, respectively at 116, 180 and 292 MeV[3].

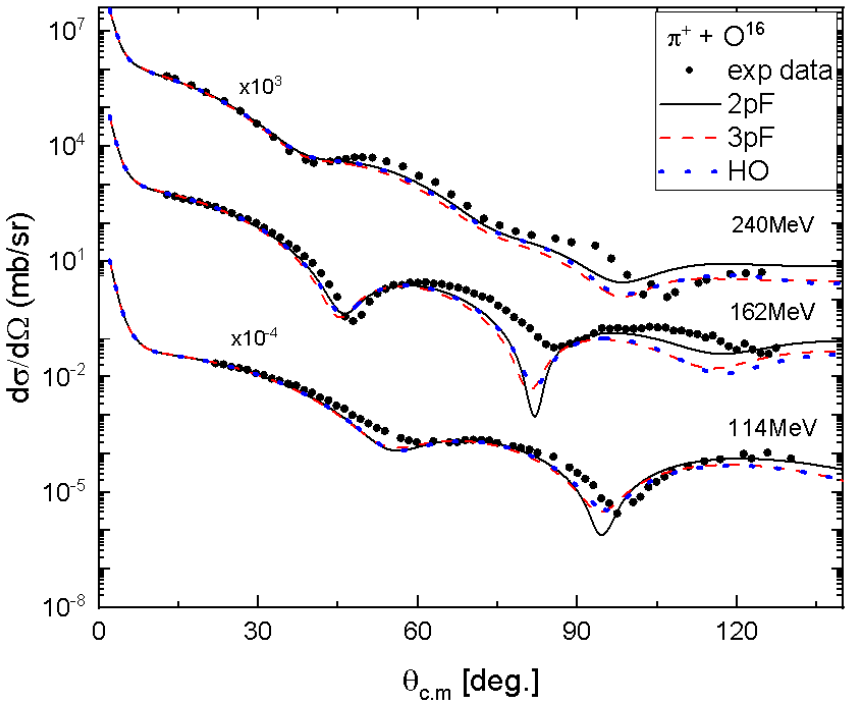


**Fig. 1** $\pi^+ + {}^{16}O$ elastic scattering at 114, 162 and 240 MeV using local Kisslinger potential with first-order parameters. The solid lines with 2pF, dashed lines with 3pF and dotted lines with HO densities for $^{16}O$. The experimental data are taken from Ref.[4].

It is shown from the three figures that elastic scattering data are not reproduced well with modified first- order parameters for all considered densities. Poor fitting for both reactions at energies below 180 MeV are found where the minimum peaks of the scattering data cannot be reproduced. By increasing energy, i.e., at 240 MeV for $\pi^+ + {}^{16}O$ and 292 MeV for $\pi^{\pm} + {}^{40}Ca$, the fitting is improved. All densities give approximately same shape indicating that the local Kisslinger potential doesn't affect by the shape of target density. It can be noticed that the fitting of $\pi^+ + {}^{40}Ca$ is much better than $\pi^- + {}^{40}Ca$ at considered energies.

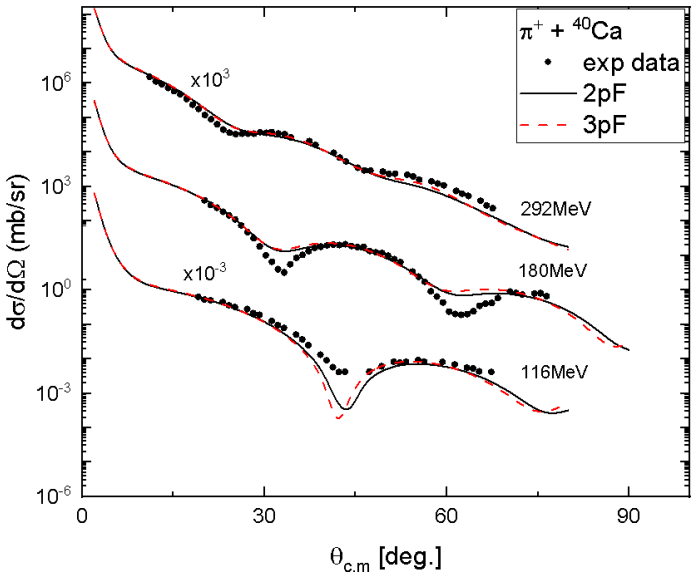


**Fig. 2** $\pi^+ + {}^{40}$Ca elastic scattering at 116, 180 and 292 MeV using local Kisslinger potential with first-order parameters. The solid lines with 2pF, dashed lines with 3pFdensities for $^{40}$Ca. The experimental data are taken from Ref.[3].

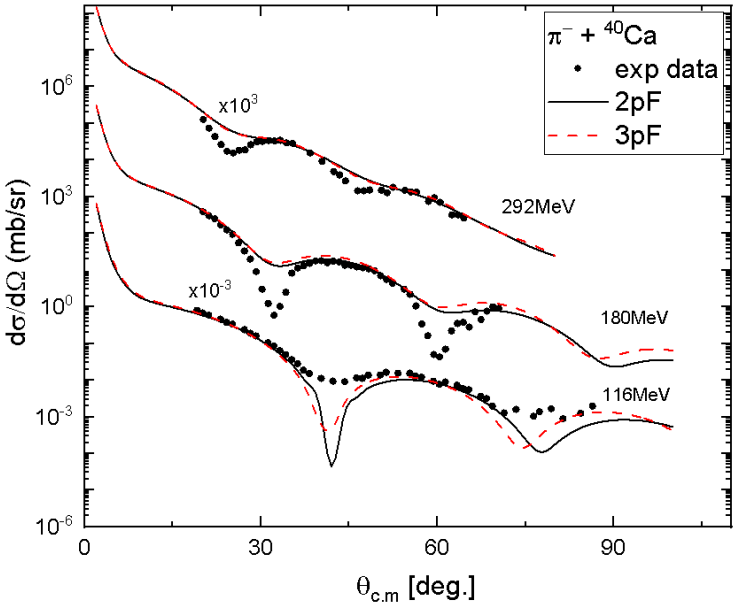


**Fig. 3** Same as Fig.2 but for $\pi^- + {}^{40}$Ca.

To confirm the importance of including second-order parameters, the local Kisslinger potential with first- plus second- order interactions are

considered for. the elastic scattering of $\pi^+ + {}^{16}O$ at 162 MeV and $\pi^{\pm} + {}^{40}Ca$ at 180 and 292 MeV. The first- plus second- order parameters are taken from Ref.[14] for $\pi^+ + {}^{16}O$ and from Ref.[17] for $\pi^{\pm} + {}^{40}Ca$. The comparison between first-order parameters only and first- plus second- order are shown in figures 4, 5 and 6. Figure 4 shows the results of $\pi^+ + {}^{16}O$ at 162 MeV while Figures 5 and 6 show the results of $\pi^+ + {}^{40}Ca$ and $\pi^- + {}^{40}Ca$, respectively at 180 and 292 MeV.

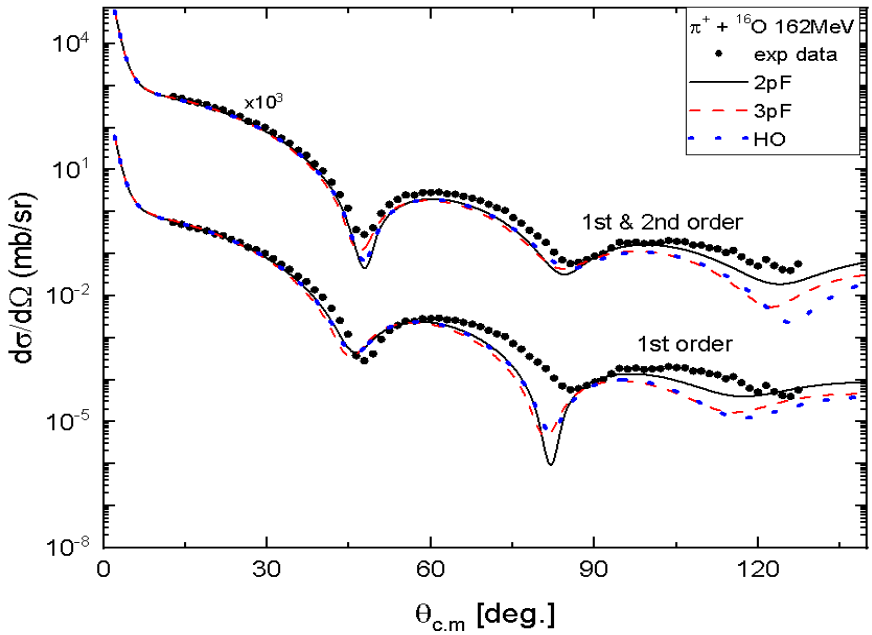


**Fig. 4** $\pi^+ + {}^{16}O$ elastic scattering at 162 MeV using local Kisslinger potential with first- plus second- order parameters and first order only. The solid lines with 2pF, dashed lines with 3pF and dotted lines with HO densities for $^{16}O$. The experimental data are taken from Ref.[4].

It is shown from the three figures that elastic scattering data are reproduced well with first- plus second- order parameters without need of any renormalization factor. Figure 4 shows that the fitting of $\pi^+ + {}^{16}O$ at 162 MeV with first- plus second- order parameters are similar to previous work[14] which means the success of our potential to reproduce same elastic scattering differential cross section. Again, all densities give approximately same differential scattering shape. So, in case of $\pi^{\pm} +$

$^{40}$Ca, we present the results of 2pF target density only. Same results are found in figures 5 and 6 where the curves with first- plus second- order parameters give best fit with experimental data even at all scattering angles in case of $\pi^+ + {}^{40}$Ca and less good for $\pi^- + {}^{40}$Ca .

## V. SUMMARY AND CONCLUSIONS

In conclusion, the local Kisslinger potential can reproduce elastic scattering data of pion-nucleus scattering with first- plus second- order parameters. The obtained results are much better than the results of first-order parameters only. Different shapes of target densities give approximately same results.

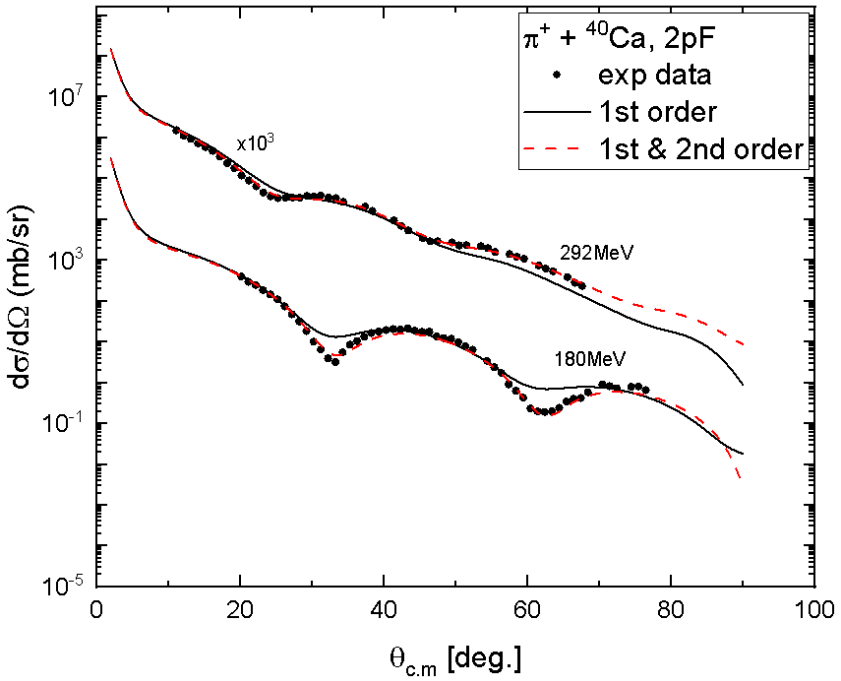


**Fig. 5** $\pi^+ + {}^{40}$Ca elastic scattering at 180 and 290 MeV using local Kisslinger potential with solid lines of first order only and dashed lines of first- plus second- order parameters. One density distribution of 2pF is considered. The experimental data are taken from Ref.[3].

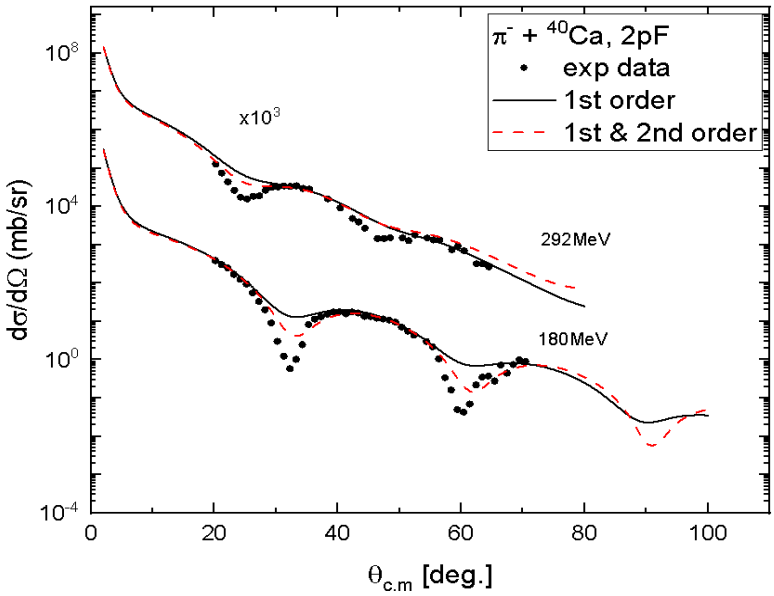


**Fig. 6** Same as Fig.5 but for $\pi^- + {}^{40}$Ca elastic scattering.

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## الملخص العربى

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نبحث فى هذه الدراسة تشتت البيونات المرن مع نواتى( $^{16}O$ ) و ( $^{40}Ca$ ) لطاقة حركية للبيونات من ١١٤ الى 292MeV. تم بناء برنامج FORTRAN لحساب local Kisslinger potential للتفاعل بين البيون و النواة للتفاعلات من الرتبة الأولى و الثانية . يأخذ الجهد المبنى فى الأعتبار جميع التصحيحات التى تمت من قبل مثل Coulomb correction و Perey factor و Ericson-Ericson Lorentez-Lorentez effect . و تدرس حساسية cross section لأشكال المختلفة من توزيع الكثافة للنواة. و أيضا تم دراسة تأثير أهمال second-order parameters . تم الحصول على نسخ جديدة من البيانات باستخدام كل من first and second-order parameters . و تم الحصول على cross section جيد مع جميع الكثافات.