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NUCLER OPTICAL MODEL SANALYSIS OF HELLION-NUCLEUS SCATTERING

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Optical potentials for the elastic scatteringof33 MeV hellion from the ^{16}O,^{24}Mg,^{32}S and ^{40}Ca target nuclei are calculated using the folding model.The real central part of the nuclear optical potential is done by folding an effective *n–α* interaction with the *α*-cluster distribution density in the target nucleus. The imaginary part of the optical potential is taken as the Woods-Saxon(WS) form. The present calculations well describe the elastic scattering of hellion from nuclei.

INTRODUCTION:

A Several studies have shown that the double folding cluster model (DFC)has been successful in analyzing a wide range of experimental data of ^{3}He scattering[1] and has been successfully reproduced the differential cross-section of elastic scattering data for some nuclear reactions [2].

On the other hand, the folding approachhas been used in the frame of the *α*-cluster modelto conclude a semi-microscopic description of the *α*-nucleus and nucleuspotentials.Previous calculations [2-5] have generated *α*-particle single folding cluster (SFC) and light HI double folding cluster (DFC) optical potentials based upon an appropriate *α-α* effective interaction. They assumed that projectile and target nuclei consist of multiple integral number of *α*-particles [3]. In addition to the α-cluster model has been successfully employed to calculate the folding optical model potential for composite projectiles through the single and double folding cluster model.[2, 4-12]. Through the last decade, El-AzabFarid and his collaborators[2, 6-8]have adopted the α-cluster structure of light nuclei to generate the α-nucleus single folding cluster (SFC) and nucleus-nucleus double folding cluster (DFC) potentials, based upon an α-α interaction folded with the α cluster distributions in the colliding nuclei as in Ref [13].

In the present work, we will focus on the interpretation of the relationship between the nuclear potential and the mass of target at laboratory energy at 33 MeV. We consider the projectile is ^{3}He and the targets are (^{16}O,^{24}Mg , ^{32}S and ^{40}Ca) and we study the angular distribution cross sections and the real and imaginary volume integral. The nuclear potential is based on α-n interaction. These reactions have been studied using the single folding cluster model SFCby A. H. Al-Ghamdiet al.[13, 14]but the potential based up on α-α interaction and showing the ability of the SFC potential to successfully reproduce the measured elastic scattering and reaction cross sections.Cook and Griffiths [15] did the same but using a double folding model using the M3Y interaction.The average value of the normalization factor for scattering at 33 MeV is N = 0.85 ± 0.06.It should be noted that the energy dependence at low energies varies greatly with target mass.

The main purpose of this workis to test the applicability of the double folding cluster modelto the elastic scattering of the nucleus ^{3}He with different nuclei at 33 MeV.The method employed here is described in section 2, the results and discussion are given in section 3, and the conclusions are presented in section 4.

2.FORMALISM:

2.1. Alpha-cluster densities

Considering the α–structure of the target nucleus with mass number $A = 4m$where m the number of α-particles. If $\rho_C(r')$is the α-clusters distribution function inside the nucleus,then we can relate the nuclear matter density distribution function of the nucleus, $\rho_M(r)$to that of the α-particle nucleus, $\rho_\alpha(r)$[2], as:

$\rho_M(r) = \int \rho_C(r')\rho_\alpha(|\vec{r} - \vec{r'}|)\, d\overrightarrow{r'}$where, (1-a)

The matter density for the target $\rho_M(r)$is taken as the harmonic form,

$\rho_M(r) = \rho_{oM}(1 + \omega r^2)\exp(-\beta r^2)$, (1-b)

with$\rho_{oM} = A[\frac{\beta}{\pi}]^{\frac{3}{2}}\,[1 + \frac{3\omega}{2\beta}]^{-1}$

The density distribution of alpha particle $\rho_\alpha(r)$,

, $\rho_\alpha(r) = \rho_{o\alpha} \exp[-\lambda r^2]$, (1-c)

with $\rho_{o\alpha} = 4[\frac{\beta}{\pi}]^{\frac{3}{2}}$

The alpha cluster distribution function $\rho_C(r')$

$\rho_c(r') = \rho_{oc}[1 + \mu r'^2] \exp[-\xi r'^2]$, (2-a)

with $\rho_{oc} = \frac{A}{32(\gamma\pi)^{\frac{3}{2}}}\ [1 + \frac{3\delta}{2\gamma}]$

, $\mu = \frac{-\delta}{4\gamma^2+6\gamma\delta}$, $\delta = \frac{-\omega}{4\beta^2\ +6\beta\omega}$, $\gamma = \frac{1}{4\beta} - \frac{1}{4\lambda}$, and $\xi = \frac{1}{4\gamma}$

The nuclear density parameters for α-particle, ^{3}He are taken from Refs.[2],[16].The parameters of the nuclear cluster densities of ^{16}O, ^{24}Mg, ^{32}S and ^{40}Ca are taken from Ref. [14].

Parameters of the nuclear cluster densities Ref. [14].

Nucleus	ρ_{0c}(fm−3)	μ (fm−2)	ζ (fm−2)	$\langle r \rangle$1/2 (fm)
^{24}Mg	0.0502	0.6002	0.3173	2.657
^{28}Si	0.0357	0.9745	0.302	2.771
^{32}S	0.0025	18.395	0.2981	2.890
^{40}Ca	0.00102	53.24	0.2903	3.048

2.2.Double folding cluster approach

The double folding cluster potential based upon α-α interaction $V_{\alpha\alpha}(\vec{r}_{12})$, folded over the fully cluster distribution density of both target and projectile nuclei denoted as DFC potential. But, in our case we consider the density for the target is fully cluster density where (^{16}O $\equiv 4\alpha$), (^{24}Mg$\equiv 6\alpha$),(^{32}S$\equiv 8\alpha$),(^{40}Ca$\equiv 10\alpha$) and the density for the projectile ^{3}He is a matter density and has a Gaussian form. So, the double folding cluster potential based upon α-n interaction $V_{\alpha n}(\vec{r}_{12})$can be considered in the following equation:

$U(\vec{r}) = \iint V_{n\alpha}(\vec{r}_{12})\rho_{cT}(\vec{r}_2)\rho_P(\vec{r}_1)d\vec{r}_1\, d\vec{r}_2$, where (3-a)

$\rho_{cT}(\vec{r}_2) = \rho_{oc}[1 + \mu\vec{r}_2{}^2]\exp[-\xi\vec{r}_2{}^2]$, and (3-b)

$\rho_p(\vec{r}_1) = \rho_o \exp[-\beta\vec{r}_1{}^2]$ (3-c)

We consider the nucleon-α interaction $V_{n\alpha}$ in Gaussian form as:

$$V_{n\alpha}(\vec{r}_{12}) = -V_o \exp\left(-\mathrm{k}\vec{r}_{12}{}^{2}\right) \tag{4}$$

Where k is the wave number, and we obtain the real part of the optical potential
through the DFC model

$$U(R) = -\pi^3 V_o \rho_{oc}\rho_o \exp\left(\frac{-\beta k R^2 f}{eg}\right)\left(\frac{1}{g^{\frac{3}{2}}} + \frac{3\mu e}{2g^{\frac{5}{2}}} + \mu\frac{(\beta k R)^2}{g^{\frac{7}{2}}}\right), \tag{5-a}$$

Where $e = \beta + k, f = \xi\beta + \xi k$, and $g = f + \beta k$ (5-b)

In the present work, the DFC potential is calculated analytically and the results are checked by recalculating the potential numerically by the computer code DOLFIN. Both methods gave identical results. The elastic scattering cross section is carried out using the computer code HIOPTIM-94.

To fit the experimental distributions, the optical model parameters were varied systematically to minimize the quantity:

$$\chi^2_\sigma = \sum_{i=1}^{N_\sigma}\left[\frac{\sigma_{th}(\theta_i) - \sigma_{ex}(\theta_i)}{\Delta\sigma_{ex}(\theta_i)}\right]^2 \tag{6}$$

Where σ_{th} and σ_{ex} are the theoretical and experimental differential cross sections, respectively,at angle θ_i , $N_\sigma$is the number of θ_i , and $\Delta\sigma_{ex}(\theta_i)$ is the error associated with$\sigma_{ex}(\theta_i)$. In this work, it is calculated numerically by a computer code HIOPTIM-94 [17].

3.RESULTS AND DISCUSSION:

The analysis of the experimental data of 33 MeV ^{3}He elastically scattered from ^{16}O, ^{24}Mg, ^{32}S and ^{40}Ca target nuclei has been carried out using the computing program HIOPTIM-94 fed with the realcentral part of the nuclear optical potential by folding an effective *n–α* interaction with the *α*-cluster distribution density in the target nuclei and the imaginary part is taken as the WS form.The radial integrations have been carriedout to a maximum radius of 40 fm in steps of 0.1 fm to account properly for Coulomb excitations.Several studies have shown that the DFC potential has successfully reproduced the differential cross-section of elastic scattering data for a few reactions [2, 3, 8, 18, 19].

We get the ^{3}He scattering data by digitizing the data in reference [15] by using the origin program,and the experimental data for ^{3}He with ^{24}Mg take from [20].

The normalization constant $N_R$can be adjusted to obtain the best fitting by minimizing the chi-square of the fitting to beclose to unity. The obtained parameters are listed in Table (1) andwe can notice from that table, the real renormalization parameter $N_R$is close to unity. The conclusion can be obtained that, there is no evidence for the renormalization N_R to be strongly dependent upon the bombarding energy in the used energy range. By using The M3Y folding model The average value of the normalization factor for elastic scattering at 33 MeV is N_R = 0.85 ±O.06. for target (^{32}S) have N_R = 0.95, substantially higher than the average value.

Figure 1 shows the angular distributions for ^{3}He elastic scattering differential cross sections from ^{24}Mg at 33 MeV calculated by theHIOPTIM-94 code using the double folding potential model compared to the experimental data. These results show that the DFC potential is able to produce a good fit as M3Y and the JLM potential see Refs.[15, 16].Figures 2-4 are the same as figure 1 but for ^{16}O, ^{32}S and ^{40}Ca target nuclei. In each figure, the solid curves show the predictions using the present potential compared to the experimental data. These figures show a reasonable agreement between measured differential cross sections and those calculated with the DFC potential with parameters given in table 1.The fits for ^{32}S nucleus are less satisfactory compared to those other nuclei.

The resulting ^{3}Hereal potentials at 33 MeV are shown in figure 5 using the DFC model. They have one feature in common; the tails of the potentials are almost identical. The interior region is poor but this region of the potential slightly participates in the elastic scattering mechanism. This result is similar to that of nucleus–nucleus [20] and pion scattering [11]. From that figure it is seen that in this case the real part is quite shallow and everywhere attractive. Thisbehavior is observed for all cases under consideration.

The reaction cross-section is considered as an important quantity in the analysis of the elastic scattering reaction. This quantity represents the probability of the loss of flux from the incident channels through nonelastic channels. In this paper, the reaction cross-section are calculated using the present calculated cluster folded potentials. Fig (8) shows in the lower panel the relation between the present calculated reaction cross-section and the target mass at the incident energy 33MeV. As it is clear from this figure, the dependence of the reaction cross-section on the target mass is linear and could be represented by the following relation,[14]

$$\sigma_R = .498\,A + 138.525 \qquad (7)$$

The present DFC optical model gives values of σ_R very close to those estimated by others [16]. This indicates that the imaginary part of the optical model listed in table (1), which is strongly correlated to σ_R, is well predicted.

The relation between the depth of the present potentials and the mass number of target nuclei is displayed in figure(7). As can be seen from this figure, the extracted depths of the present real potentials decay rapidly as the mass number of target nuclei increases.

The volume integral represents the strength of the considered potential. For the imaginary potential this quantity is calculated by the following relation,[14]

$$J_I = \frac{1}{4A_T}\int W(R)\,dR \qquad (8)$$

The upper part of figure(8) represents the change of this quantity with target mass at incident energy 33 MeV. From this figure, it is shown the imaginary volume integral increase with increasing the target mass. This mass dependence could be represented the following relation,

$$J_I = 19.313\,A + 1152.5 \qquad (9)$$

From figure(7) , figure(8) and table 1, it is seen that both the depth of the real part of the optical potential V_o and the corresponding volume integral of the target nucleus J_R decrease with the increase of the mass number of the target nucleus. This behavior was also noticed for ^{3}He–nucleus scattering[21] .

4.CONCLUSION:

We used the present real DFC optical potential with the imaginary WS formto obtain a good description for ^{3}He elastic scattering from ^{16}O, ^{24}Mg, ^{32}S and ^{40}Ca target nuclei at 33 MeV with HIOPTIM-94code. It seems that the present calculationspredict well the maximum and minimum positions at large angles.Also, the present optical model calculation gives reaction cross sections σ_R, as listed in Table (1), in reasonable agreement with the other calculations [15, 16].

Finally, it may be concluded that the double folding cluster model potential has the ability to reproduce the shape and magnitude of the^{3}He elastic scattering data from ^{16}O, ^{24}Mg, ^{32}S and ^{40}Ca target nuclei at an incident energy 33 MeV.

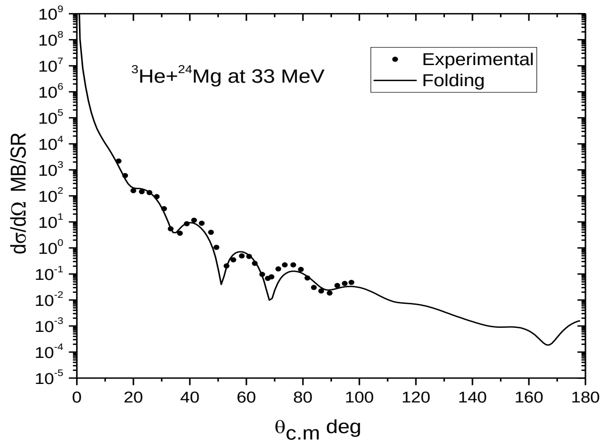

Figure 1. The differential cross sections for ^{3}He with ^{24}Mg elastic scattering at 33 MeV

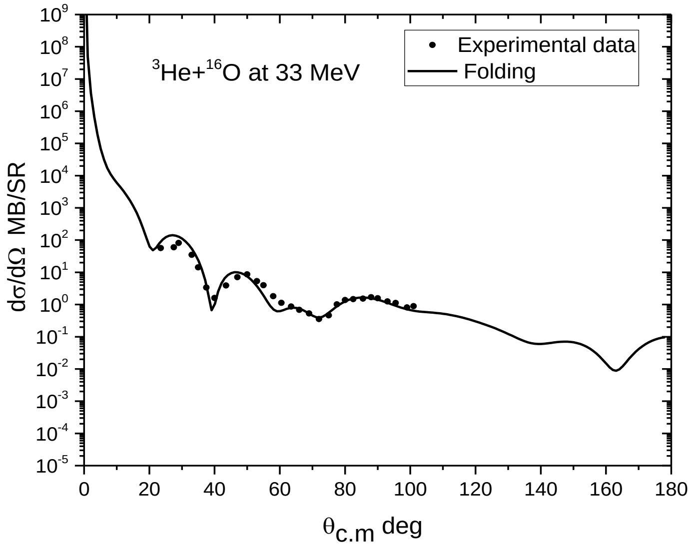

Figure 2. The differential cross sections for ^{3}He with ^{16}O elastic scattering at 33 MeV

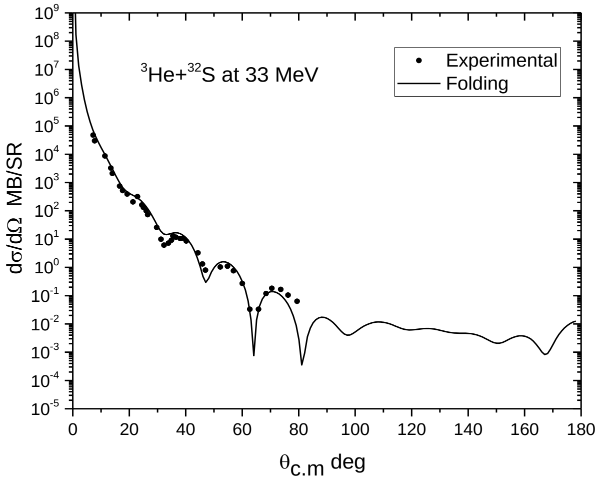

Figure 3. The differential cross sections for ^{3}He with ^{32}S elastic scattering at 33 MeV

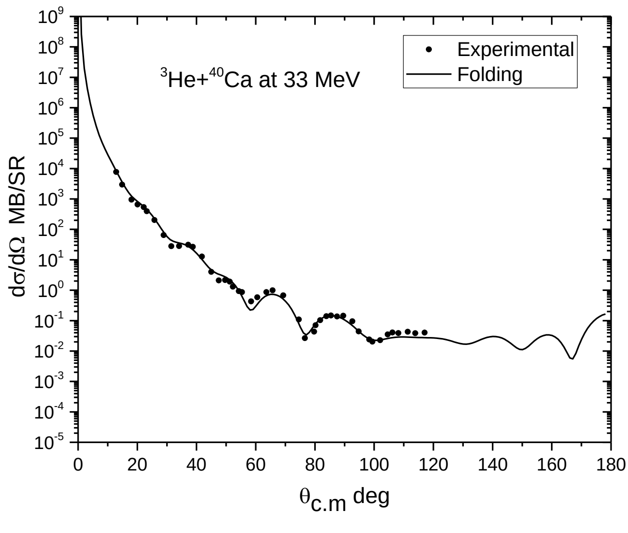

Figure 4. The differential cross sections for ^{3}He with ^{40}Ca elastic scattering at 33 MeV

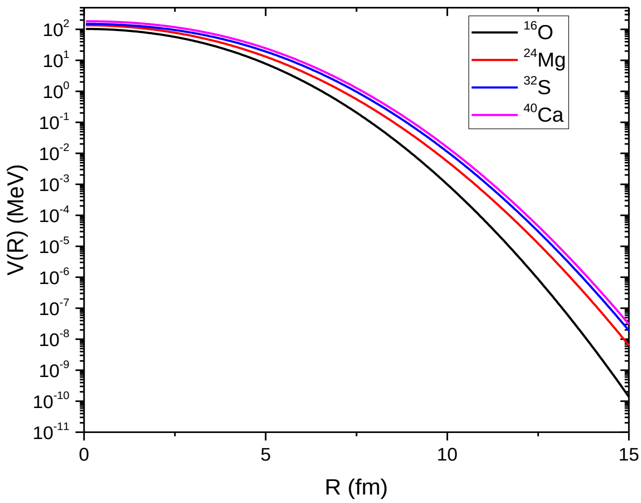

Figure 5. The real part of the potential calculated using the DFC density at all energies (logarithmic scale).

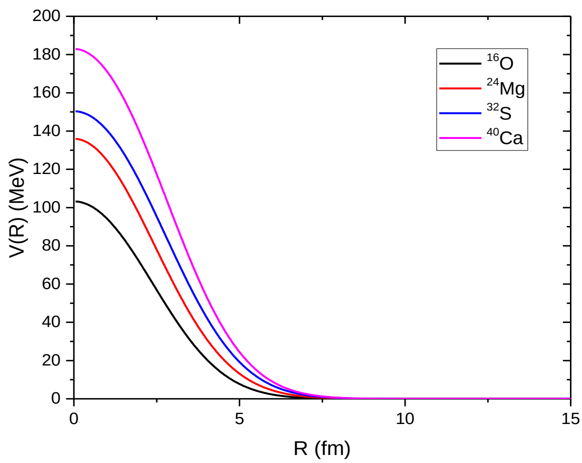

Figure 6. The real part of the potential calculated using the DFC density at all energies (linear scale).

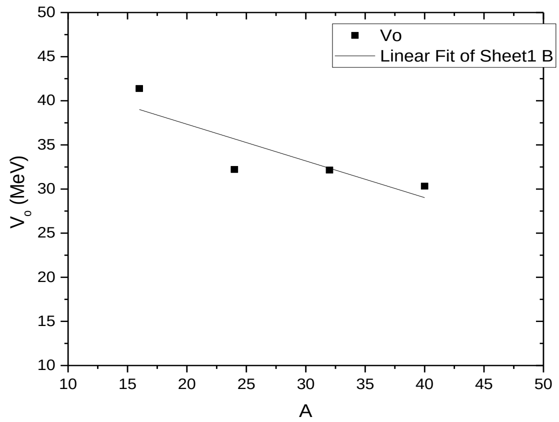

Figure 7. The relation between V_o and A for ^{3}He with different target elastic scattering

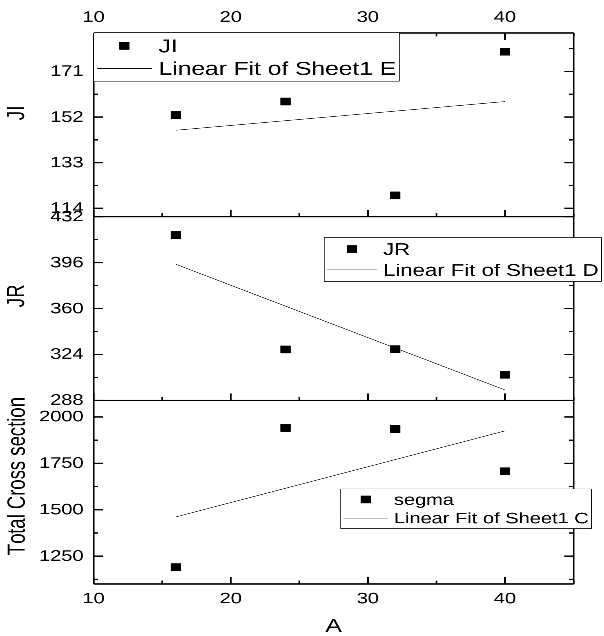

Figure 8. The relation between the total cross section ,volume integral for real part and volume integral for the imaginary part with the mass number (A) for ^{3}He with different target elastic scattering

Table(1): The parameters of the elastic scattering for ^{3}He with different target at energy 33 MeV

Target	P	US	WS	WD	RI	AI	χ^2	σ_R	JR	JI
^{16}O	F	1.14	_____	12.9	1.597	.6441	19.4	1.2E+3	417.6	152.9
^{32}S	F	0.89	26.42	_____	1.305	1.272	27.4	1.94E+03	327.9	158.5
^{40}Ca	F	0.88	10.93	_____	1.835	1.031	7.23	1.94E+03	328.1	119.3
^{24}Mg exfor	F	0.8	26.23	____	1.39	1.14	17.6	1.71E+03	308.11	179.21

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تحليل النموذج الضوئى النووى لاستطارة نواة الهيليون

شريف رشاد مختار ، زكريا محمد محمد محمود وهند سلامة عبد العال
قسم الفيزياء – كلية العلوم - جامعة أسيوط

لقد تم حساب الجهود الضوئية للتشتت المرن عند الطاقة ٣٣ (م إ ف) للهليون من الاكسجين ١٦ ، المغنسيوم ٢٤، الكبريت ٣٢ والكالسيوم ٤٠ كانوية هدف باستخدام نموذج الطي .
ولقد تم حساب الجزء الحقيقي المركزي للجهد الضوئي النووي عن طريق طي الجهد الفعال(n-α) مع كثافة انوية الهدف ذات التوزيع العنقودي . اما الجزء التخيلي اخذ بشكل Wood-Saxon .
الحسابات الحالية تصف جيدا التشتت المرن للهليون من الانوية .