Assiut Univ. J. of Mathematics and Computer Science Printed ISSN 1687-4897
49(2), pp.52-70 (2020)

SOFT COMPACTNESS OF FUZZY SOFT PRETOPOLOGICAL SPACES

F. H. Khedr, M. Azab. Abd－Allah, E. A. Abdelgaber
Mathematics department, Faculty of science, Assuit university, Assuit
Khedrfathi@gmail.com
Mathematics department, Faculty of science, Assuit university, Assuit
Mazab57@yahoo.com
Mathematics department, Faculty of science, Minia university, Minia
eman1357977@yahoo.com

Received: 6/10/2020 **Accepted:** 18/11/2020 **Available Online:** 1/12/2020

The notions of soft $1-$compactness, soft $2-$compactness and some of weaker forms of soft compactness such as soft locally compactness, soft countable compactness and soft lindelöf are introduced and studied in fuzzy soft pretopological spaces.

Keywords: The $\alpha-$finite intersection property; soft compact; degree of soft non$-$ vacuity; soft locally compact; soft lindelöf and soft countable compact.

1. INTRODUCTION

Maji et al. in [7] introduced the concept of a fuzzy soft set and some properties. Tany and Kandemir [9] gave the notion of a fuzzy soft topology. In [1], Atmaca et al. introduced the concept of a fuzzy soft function. Brissaud in [3] introduced the concept of a pretopology. The concepts of a fuzzy pretopology, $1-$ compactness and $2-$compactness were introduced and studied by Badard in [2]. Mashour et al. in [8] introduced and studied the concepts of countable compactness and lindelöf in $L-$fuzzy pretoological spaces. Khedr et al. in [6] introduced the definition of a fuzzy soft pretopology. Some important properties of fuzzy soft pretopological spaces were discussed and studied.

The aim of this paper is to introduce the dual of ψ, soft $a-$cover, soft compactness, soft lindelöf and soft countable compactness with some properties. Also, the concepts of soft weakly locally compactness and soft locally compactness are introduced and studied.

2. PRELIMINARIES

Definition 2.1 [7]. Let X be an initial universe set, E be a set of parameters, I^X denotes the set of all fuzzy sets of X and $A \subseteq E$. A pair (F,A), denoted by F_A, is called a fuzzy soft set over (X,E) if F is a mapping given by $F:A\to I^X$ such that $F(e)\neq 0_X$ if $e\in A$ and $F(e)=0_X$ if $e\notin A$, where $0_X(x)=0$ for all $x\in X$. Thus a fuzzy soft set F_A over (X,E) can be represented by the set of ordered pairs $F_A=\{(e,F_A):e\in A,F_A\in I^X\}$. In other words, the fuzzy soft set is a parameterized family of fuzzy subsets of the set X.

The set of all fuzzy soft sets over an initial universe set X and a set of parameters E is denoted by $FSS(X,E)$.

Definition 2.2 [1]. A fuzzy soft set F_A over (X,E) is said to be a fuzzy soft point if there is an $e\in E$ such that $F_A(e)$ is a fuzzy point in X (i.e., there exists an $x\in X$ such that $F_A(e)(x)=\alpha\in\left]0,1\right]$ and $F_A(e)(x')=0$ for all $x'\in X-\{x\}$) and $F_A(e')=0_X$ for every $e'\in E-\{e\}$. It will be denoted by x_e^α. The fuzzy soft point x_e^α is said to be belongs to a fuzzy soft set F_A, denoted by $x_e^\alpha\tilde{\in}F_A$, if $\alpha\leq F_A(e)(x)$.

Definition 2.3 [6]. A fuzzy soft pretopology on (X,E) is a function $a:FSS(X,E)\to FSS(X,E)$ which satisfies the following conditions:-

$(PT1)$ $a(\tilde{\phi})=\tilde{\phi}$.

$(PT2)$ $a(F_A)\tilde{\supseteq}F_A$, for every $F_A\tilde{\in}FSS(X,E)$.

The triple (X,E,a) is then said to be a fuzzy soft pretopological space.

A fuzzy soft pretopological space (X,E,a) is said to be of:-

$(PT3)$ Type I: if for every $F_A,\ G_B\tilde{\in}FSS(X,\ E)$ such that $F_A\tilde{\subseteq}G_B$, we have $a(F_A)\tilde{\subseteq}a(G_B)$.

$(PT4)$ Type D: if for every $F_A,G_B\tilde{\in}FSS(X,\ E)$, we have $a(F_A\tilde{\cup}G_B)=a(F_A)\tilde{\cup}a(G_B)$.

$(PT5)$ Type S: if for every $F_A\tilde{\in}FSS(X,\ E)$, we have $a^2(F_A)=a(a(F_A))=a(F_A)$.

We say that a is of type $\sigma\eta$ if a is of type σ and η, where $\sigma,\eta\in\{I,D,S\}$.

One notice that $(PT\,4) \Rightarrow (PT\,3)$ and any fuzzy soft pretopological space (X, E, a) of type DS is a fuzzy soft topological space where a is Kuratowski closure.

Definition 2.4 [6]. Let a_1, a_2 be two fuzzy soft pretopologies on (X, E). a_1 is said to be coarser than a_2, written $a_1 \tilde{\leq} a_2$ if $a_1(F_A) \tilde{\subseteq} a_2(F_A)$, for every $F_A \tilde{\in} FSS(X, E)$.

Definition 2.5 [6]. Let a be a fuzzy soft pretopology, the fuzzy soft interior function $i_a : FSS(X, E) \rightarrow FSS(X, E)$ is defined by $i_a(F_A) = (a(F_A^c))^c$.

The function i_a satisfies the following properties:-

$(PT\,1)$ $i_a(\tilde{\phi}) = \tilde{\phi}$;

$(PT\,2)$ For every $F_A \tilde{\in} FSS(X, E)$, we have $i_a(F_A) \tilde{\subseteq} F_A$;

$(PT\,3)$ For every F_A, $G_B \tilde{\in} FSS(X, E)$ such that $F_A \tilde{\subseteq} G_B$, we have $i_a(F_A) \tilde{\subseteq} i_a(G_B)$;

$(PT\,4)$ For every $F_A, G_B \tilde{\in} FSS(X, E)$, we have $i_a(F_A \tilde{\cap} G_B) = i_a(F_A) \tilde{\cap} i_a(G_B)$;

$(PT\,5)$ For every $F_A \tilde{\in} FSS(X, E)$, we have $i_a^2(F_A) = F_A$.

Definition 2.6 [6]. Let a be a fuzzy soft pretopology and $F_A \tilde{\in} FSS(X, E)$. Then F_A is said to be a fuzzy soft pre−open (resp. fuzzy soft pre−closed) set in (X, E) if $i_a(F_A) = F_A$ (resp. $a(F_A) = F_A$).

It is clear that F_A is a fuzzy soft pre−closed set if and only if F_A^c is fuzzy soft pre−open.

Definition 2.7 [6]. Let a be a fuzzy soft pretopology and $F_A \tilde{\in} FSS(X, E)$. The trace of a on F_A, denoted by a_{F_A}, is defined as follows: $a_{F_A}(G_B) = a(F_A) \tilde{\cap} G_B$, for every fuzzy soft subset G_B of F_A.

Definition 2.8 [6]. A function $\psi : FSS(X, E) \rightarrow I$ is said to be a degree of soft non−vacuity if it satisfies:-

(1) $\psi(\tilde{\phi}) = 0$;

(2) $\psi(G_B)=1$ if there exists a soft point x_e such that $\mu_{G_B}(x_e)=1$;

(3) If $G_B \tilde{\subseteq} H_C$, then $\psi(G_B)\le\psi(H_C)$.

In particular, $\psi(G_B)=\sup_{x_e\tilde{\in}(X,E)}\mu_{G_B}(x_e)$ is a degree of soft non $-$ vacuity.

Definition 2.9 [4]. A family ξ of fuzzy soft sets is said to be a cover of a fuzzy soft set F_A if $F_A \tilde{\subseteq} \tilde{\bigcup}_{j\in J}\{(F_A)_j : (F_A)_j \tilde{\in} \xi, j\in J\}$. A subcover of ξ is a subfamily of ξ which is also a cover.

Definition 2.10 [4]. A family ξ of fuzzy soft sets is said to be satisfies the finite intersection property (FIP) if the intersection of the members of each finite subfamily of ξ is not null fuzzy soft set.

Definition 2.11 [5]. Let $FSS(X,E_1)$ and $FSS(Y,E_2)$ be the families of all fuzzy soft sets over (X,E_1) and (Y,E_2) respectively. Let $u: X\to Y$ and $p: E_1\to E_2$ be two functions. The fuzzy soft function from $FSS(X,E_1)$ to $FSS(Y,E_2)$, denoted by $f_{up}: FSS(X,E_1)\to FSS(Y,E_2)$, is defined as follows:

(1) Let F_A be a fuzzy soft set in $FSS(X,E_1)$. Then the image of F_A under the fuzzy soft function f_{up} is the fuzzy soft set over (Y,E_2), denoted by $f_{up}(F_A)$, where

$$f_{up}(F_A)(e_2)(y)=\begin{cases}\bigvee_{x\in u^{-1}(y)}\left(\bigvee_{e_1\in p^{-1}(e_2)\cap A}F(e_1)\right)(x), & \text{if } u^{-1}(y)\neq\tilde{\phi}, p^{-1}(e_2)\cap A\neq\phi\\ 0_Y, & \text{otherwise}\end{cases}$$

(2) Let G_B be a fuzzy soft set in $FSS(Y,E_2)$. Then the preimage (inverse image) of G_B under the fuzzy soft function f_{up} is the fuzzy soft set over (X,E_1), denoted by $f_{up}^{-1}(G_B)$, where

$$f_{up}^{-1}(G_B)(e_1)(x)=\begin{cases}G_B(p(e_1))(u(x)), & \text{for } p(e_1)\in B\\ 0_X, & \text{otherwise}\end{cases}$$

If u and p are injective (resp. surjective), then the fuzzy soft function f_{up} is said to be injective (resp. surjective).

Theorem 2.1 [5]. Let $F_A \tilde{\in} FSS(X, E_1)$, $\{(F_A)_j : j \in J\} \tilde{\subset} FSS(X, E_1)$, $G_B \tilde{\in} FSS(Y, E_2)$ and $\{(G_B)_j : j \in J\} \tilde{\subset} FSS(Y, E_2)$ where J is an index set. Then:-

(1) If $(F_A)_1 \tilde{\subseteq} (F_A)_2$, then $f_{up}(F_A)_1 \tilde{\subseteq} f_{up}(F_A)_2$.

(2) If $(G_B)_1 \tilde{\subseteq} (G_B)_2$, then $f_{up}^{-1}(G_B)_1 \tilde{\subseteq} f_{up}^{-1}(G_B)_2$.

(3) $f_{up}(\tilde{\bigcap}_{j \in J} (F_A)_j) \tilde{\subseteq} \tilde{\bigcap}_{j \in J} f_{up}(F_A)_j$, and the equality holds if f_{up} is injective.

(4) $f_{up}^{-1}(\tilde{\bigcap}_{j \in J} (G_B)_j) = \tilde{\bigcap}_{j \in J} f_{up}^{-1}(G_B)_j$.

(5) $F_A \tilde{\subseteq} f_{up}^{-1}(f_{up}(F_A))$, and the equality holds if f_{up} is injective.

(6) $f_{up}(f_{up}^{-1}(G_B)) \tilde{\subseteq} G_B$, and the equality holds if f_{up} is surjective.

(7) $f_{up}^{-1}(\tilde{Y}) = \tilde{X}$, $f_{up}^{-1}(\tilde{\phi}) = \tilde{\phi}$ and $f_{up}(\tilde{\phi}) = \tilde{\phi}$.

(8) $f_{up}(\tilde{X}) \tilde{\subseteq} \tilde{Y}$, and the equality holds if f_{up} is surjective.

3. SOFT COMPACTNESS AND SOFT LOCALLY COMPACTNESS

Definition 3.1. A family $\{(F_A)_j : j \in J\}$ of fuzzy soft subsets of (X, E) is said to be satisfies the $\alpha-$finite intersection property if $\psi(\tilde{\bigcap}_{j \in J_o} (F_A)_j) \geq \alpha$, for every finite subset J_o of J.

Definition 3.2. Let (X, E, a) be a fuzzy soft pretopological space of type I, then (X, E) is said to be soft $1-$compact (resp. soft $2-$compact) if for every family $\{(F_A)_j : j \in J\}$ of fuzzy soft subsets of (X, E) which verifies the finite intersection property (resp. the $\alpha-$finite intersection property), we can assert that $\tilde{\bigcap}_{j \in J} a(F_A)_j \neq \tilde{\phi}$ (resp. $\psi(\tilde{\bigcap}_{j \in J} a(F_A)_j) \geq \alpha$).

It is clear that the classical definition for fuzzy soft compactness is a special case of soft $1-$compactness because the family $\{(F_A)_j : j \in J\}$ is taken with $(F_A)_j = a(F_A)_j$ and then $\tilde{\bigcap}_{j\in J} a(F_A)_j = \tilde{\bigcap}_{j\in J}(F_A)_j$.

Definition 3.3. Let (X, E, a) be a fuzzy soft pretopological space of type I and G_B a fuzzy soft subset of (X, E). Then G_B is said to be soft $1-$compact (resp. soft $2-$compact) if for every family $\{(F_A)_j : j \in J\}$ and $(F_A)_j \tilde{\subseteq} G_B$ which verifies the finite intersection property (resp. the $\alpha-$finite intersection property), we can assert that $\tilde{\bigcap}_{j\in J} a_{G_B}(F_A)_j \neq \tilde{\phi}$ (resp. $\psi(\tilde{\bigcap}_{j\in J} a_{G_B}(F_A)_j) \geq \alpha$).

Proposition 3.1. Every fuzzy soft pre$-$closed subset of a soft $1-$compact fuzzy soft pretopological space is soft $1-$compact.

Proof. Let (X, E, a) be a soft $1-$compact fuzzy soft pretopological space, G_B be a fuzzy soft pre$-$closed subset of (X, E) and $\{(F_A)_j : j \in J\}$, $(F_A)_j \tilde{\subseteq} G_B$ be a family of fuzzy soft subsets which verifies the finite intersection property. Then $\tilde{\bigcap}_{j\in J} a(F_A)_j \neq \tilde{\phi}$. But $(F_A)_j \tilde{\subseteq} G_B$ implies $a(F_A)_j \tilde{\subseteq} a(G_B) = G_B$.

Now, $\tilde{\bigcap}_{j\in J} a_{G_B}(F_A)_j = \tilde{\bigcap}_{j\in J} a(F_A)_j \tilde{\cap} G_B = \tilde{\bigcap}_{j\in J} a(F_A)_j \neq \tilde{\phi}$ which proves that G_B is soft $1-$compact.

Proposition 3.2. Every fuzzy soft pre$-$closed subset of a soft $2-$compact fuzzy soft pretopological space is soft $2-$compact.

Proof. Suppose that (X, E, a) is a soft $2-$compact fuzzy soft pretopological space, G_B be a fuzzy soft pre$-$closed subset of (X, E) and $\{(F_A)_j : j \in J\}$, $(F_A)_j \tilde{\subseteq} G_B$ be a family of fuzzy soft subsets which verifies the $\alpha-$finite intersection property. Then $\psi(\tilde{\bigcap}_{j\in J} a(F_A)_j) \geq \alpha$. But $(F_A)_j \tilde{\subseteq} G_B$ implies $a(F_A)_j \tilde{\subseteq} a(G_B) = G_B$.

Now, $\psi(\tilde{\bigcap}_{j\in J} a_{G_B}(F_A)_j) = \psi(\tilde{\bigcap}_{j\in J} a(F_A)_j \tilde{\cap} G_B) = \psi(\tilde{\bigcap}_{j\in J} a(F_A)_j) \geq \alpha$ which proves that G_B is soft $2-$compact.

Definition 3.4. A function $\tilde{\psi}: FSS(X,E) \to I$ is said to be the dual of ψ, where ψ is a degree of soft non−vacuity if it verifies: $\tilde{\psi}(F_A) = (\psi(F_A^c))^c$ or equivalently $\psi(F_A) = (\tilde{\psi}(F_A^c))^c$.

Definition 3.5. Let (X,E,a) be a fuzzy soft pretopological space. A family $\{(F_A)_j : j \in J\}$ of fuzzy soft subsets of (X,E) is said to be soft $a-$cover of (X,E) if $\{i_a(F_A)_j : j \in J\}$ covers (X,E).

Theorem 3.1. A fuzzy soft pretopological space (X,E,a) of type I is soft $1-$compact if and only if for every soft $a-$cover $\{(F_A)_j : j \in J\}$ of (X,E), there exists a finite subset J_o of J such that $\{(F_A)_j : j \in J_o\}$ covers (X,E).

Proof. Suppose that $\{(F_A)_j : j \in J\}$ is a soft $a-$cover of (X,E) and there is no finite subset J_o of J such that $\{(F_A)_j : j \in J_o\}$ covers (X,E). Then $\tilde{\bigcap}_{j \in J_o} (F_A)_j^c \neq \tilde{\phi}$, for every finite subset J_o of J. Since (X,E,a) is soft $1-$compact, then $\tilde{\bigcap}_{j \in J} a(F_A)_j^c \neq \tilde{\phi}$. Therefore, $\{i_a(F_A)_j : j \in J\}$ does not cover (X,E) which contradicts our assumption. Conversely, let $\{(F_A)_j : j \in J\}$ be a family of fuzzy soft subsets of (X,E) such that $\tilde{\bigcap}_{j \in J_o} (F_A)_j \neq \tilde{\phi}$, for every finite subset $J_\circ$ of J and (X,E,a) is not soft $1-$compact. Then $\tilde{\bigcap}_{j \in J} a(F_A)_j = \tilde{\phi}$ which implies that $\{(F_A)_j^c : j \in J\}$ is a soft $a-$cover of (X,E). Thus there exists a finite subset J_o of J such that $\{(F_A)_j^c : j \in J_o\}$ covers (X,E). Therefore, $\tilde{\bigcap}_{j \in J_o} (F_A)_j = \tilde{\phi}$ which is a contradiction. Thus (X,E,a) is soft $1-$compact.

Theorem 3.2. A fuzzy soft pretopological space (X,E,a) of type I is soft $2-$compact if and only if for every family $\{(F_A)_j : j \in J\}$ of fuzzy soft subsets of

(X,E) such that $\tilde{\psi}(\underset{j\in J}{\tilde{\cup}} i_a(F_A)_j)\not\leq \alpha'$, there exists a finite subset J_o of J such that $\tilde{\psi}(\underset{j\in J_o}{\tilde{\cup}}(F_A)_j)\not\leq \alpha'$.

Proof. Let $\{(F_A)_j : j\in J\}$ be a family of fuzzy soft subsets of (X,E) such that $\tilde{\psi}(\underset{j\in J}{\tilde{\cup}} i_a(F_A)_j)\not\leq \alpha'$ and $\tilde{\psi}(\underset{j\in J_o}{\tilde{\cup}}(F_A)_j)\leq \alpha'$, for every finite subset J_o of J. So, $\psi(\underset{j\in J_o}{\tilde{\cap}}(F_A)_j^c)\geq \alpha$. But (X,E,a) is soft $2-$compact which implies that $\psi(\underset{j\in J}{\tilde{\cap}} a(F_A)_j^c)\geq \alpha$. So, $\tilde{\psi}(\underset{j\in J}{\cup} i_a(F_A)_j)\leq \alpha'$ which is a contradiction. Thus $\tilde{\psi}(\underset{j\in J_o}{\tilde{\cup}}(F_A)_j)\not\leq \alpha'$. Conversely, suppose that for every family $\{(F_A)_j : j\in J\}$ of fuzzy soft subsets of (X,E) such that $\tilde{\psi}(\underset{j\in J}{\tilde{\cup}} i_a(F_A)_j)\not\leq \alpha'$, there exists a finite subset J_o of J such that $\tilde{\psi}(\underset{j\in J_o}{\tilde{\cup}}(F_A)_j)\not\leq \alpha'$. Assume that (X,E,a) is not soft $2-$compact and $\psi(\underset{j\in J_o}{\tilde{\cap}}(F_A)_j)\geq \alpha$, for every finite subset $J_\circ$ of J. Then $\psi(\underset{j\in J}{\tilde{\cap}} a(F_A)_j)\not\geq \alpha$. So, $\tilde{\psi}(\underset{j\in J}{\tilde{\cup}} i_a(F_A)_j^c)\not\leq \alpha'$, then there exists a finite subset J_o^* of J such that $\tilde{\psi}(\underset{j\in J_o^*}{\tilde{\cup}}(F_A)_j^c)\not\leq \alpha'$. Since $\psi(\underset{j\in J_o}{\tilde{\cap}}(F_A)_j)\geq \alpha$, then $\tilde{\psi}(\underset{j\in J_o}{\tilde{\cup}}(F_A)_j^c)\leq \alpha'$ which is a contradiction. Thus (X,E,a) is soft $2-$compact.

Proposition 3.3. Let a_1, a_2 be fuzzy soft pretopologies on (X,E), $a_1 \tilde{\leq} a_2$ and (X,E,a_1) is soft $1-$compact (resp. soft $2-$compact), then (X,E,a_2) is soft $1-$compact (resp. soft $2-$compact) .

Proof. (1) Let $a_1 \tilde{\leq} a_2$, (X,E,a_1) be a soft $1-$compact fuzzy soft pretopological space and $\{(F_A)_j : j\in J\}$ be a family of fuzzy soft subsets of (X,E) which satisfies the finite intersection property. Then $\underset{j\in J}{\tilde{\cap}} a_1(F_A)_j \neq \tilde{\phi}$. Since $a_1 \tilde{\leq} a_2$, then $a_1(F_A)_j \tilde{\subseteq} a_2(F_A)_j$, for every

$(F_A)_j \tilde{\in} FSS(X,E)$ and so $\tilde{\bigcap}_{j\in J} a_1(F_A)_j \tilde{\subseteq} \tilde{\bigcap}_{j\in J} a_2(F_A)_j$. But $\tilde{\bigcap}_{j\in J} a_1(F_A)_j \neq \tilde{\phi}$, then we have $\tilde{\bigcap}_{j\in J} a_2(F_A)_j \neq \tilde{\phi}$ which proves that (X,E,a_2) is soft $1-$compact.

(2) Let $a_1 \tilde{\leq} a_2$, (X,E,a_1) be a soft $2-$compact fuzzy soft pretopological space and $\{(F_A)_j : j \in J\}$ be a family of fuzzy soft subsets of (X,E) which satisfies the $\alpha-$finite intersection property. Then $\psi(\tilde{\bigcap}_{j\in J} a_1(F_A)_j) \geq \alpha$. Since $a_1 \tilde{\leq} a_2$, then $a_1(F_A)_j \tilde{\subseteq} a_2(F_A)_j$, for every $(F_A)_j \tilde{\in} FSS(X,E)$ and so $\psi(\tilde{\bigcap}_{j\in J} a_1(F_A)_j) \leq \psi(\tilde{\bigcap}_{j\in J} a_2(F_A)_j)$. But $\psi(\tilde{\bigcap}_{j\in J} a_1(F_A)_j) \geq \alpha$, then we have $\psi(\tilde{\bigcap}_{j\in J} a_2(F_A)_j) \geq \alpha$ which proves that (X,E,a_2) is soft $2-$compact.

Definition 3.6. Let (X,E_1,a) and (Y,E_2,b) be two fuzzy soft pretopological spaces. A fuzzy soft function $f_{up} : (X,E_1,a) \to (Y,E_2,b)$ is said to be:-

(1) Fuzzy soft pre$-$continuous if $f_{up}(a(F_A)) \tilde{\subseteq} b(f_{up}(F_A))$, for every $F_A \tilde{\in} FSS(X,E)$.

(2) Fuzzy soft pre$-$open if $f_{up}(i_a(F_A)) \tilde{\subseteq} i_b(f_{up}(F_A))$, for every $F_A \tilde{\in} FSS(X,E)$.

(3) Fuzzy soft pre$-$closed if $f_{up}(a(F_A)) \tilde{\supseteq} b(f_{up}(F_A))$, for every $F_A \tilde{\in} FSS(X,E)$.

Proposition 3.4. Let (X,E_1,a) and (Y,E_2,b) be two fuzzy soft pretopological spaces of type I and $f_{up} : (X,E_1,a) \to (Y,E_2,b)$ be a fuzzy soft bijective pre$-$continuous function. If G_B is a soft $1-$compact fuzzy soft subset of (X,E_1), then $f_{up}(G_B)$ is a soft $1-$compact subset of (Y,E_2).

Proof. Let $\{(F_A)_j : j \in J\}$, $(F_A)_j \tilde{\subseteq} f_{up}(G_B)$ be a family of fuzzy soft subsets of (Y,E_2) which verifies the finite intersection property. Note that

$(f_{up}^{-1})_{G_B}(F_A)_j$ is the fuzzy soft subset $f_{up}^{-1}(F_A)_j\,\tilde{\cap}G_B$. For simplisity we use a^* for a_{G_B} and b^* for $b_{f_{up}(G_B)}$. Since $(F_A)_j\,\tilde{\subseteq}f_{up}(G_B)$, then $f_{up}^{-1}(F_A)_j\,\tilde{\subseteq}f_{up}^{-1}(f_{up}(G_B))=G_B$ because f_{up} is fuzzy soft injective. Now, $\underset{j\in J_o}{\tilde{\cap}}(f_{up}^{-1})_{G_B}(F_A)_j=\underset{j\in J_o}{\tilde{\cap}}f_{up}^{-1}(F_A)_j\,\tilde{\cap}G_B=\underset{j\in J_o}{\tilde{\cap}}f_{up}^{-1}(F_A)_j=f_{up}^{-1}(\underset{j\in J_o}{\tilde{\cap}}(F_A)_j)\neq\tilde{\phi}$ because f_{up} is fuzzy soft surjective. Therefore, $(f_{up}^{-1})_{G_B}(F_A)_j$ satisfy the finite intersection property. Since G_B is soft $1-$compact, then $\underset{j\in J}{\tilde{\cap}}a^*((f_{up}^{-1})_{G_B}(F_A)_j)\neq\tilde{\phi}$. Since f_{up} is fuzzy soft pre$-$continuous, then

$$\begin{aligned}f_{up}(\underset{j\in J}{\tilde{\cap}}a^*((f_{up}^{-1})_{G_B}(F_A)_j))&\tilde{\subseteq}\underset{j\in J}{\tilde{\cap}}f_{up}(a^*((f_{up}^{-1})_{G_B}(F_A)_j))\\&=\underset{j\in J}{\tilde{\cap}}f_{up}(a((f_{up}^{-1})_{G_B}(F_A)_j)\,\tilde{\cap}G_B)\\&\tilde{\subseteq}\underset{j\in J}{\tilde{\cap}}(f_{up}(a((f_{up}^{-1})_{G_B}(F_A)_j))\,\tilde{\cap}f_{up}(G_B))\\&\tilde{\subseteq}\underset{j\in J}{\tilde{\cap}}f_{up}(a((f_{up}^{-1})_{G_B}(F_A)_j))=\underset{j\in J}{\tilde{\cap}}f_{up}(a(f_{up}^{-1}(F_A)_j\,\tilde{\cap}G_B))\\&=\underset{j\in J}{\tilde{\cap}}f_{up}(a(f_{up}^{-1}(F_A)_j))\,\tilde{\subseteq}\underset{j\in J}{\tilde{\cap}}b(f_{up}(f_{up}^{-1}(F_A)_j))=\underset{j\in J}{\tilde{\cap}}b(F_A)_j\end{aligned}$$

Therefore, $f_{up}(\underset{j\in J}{\tilde{\cap}}a^*((f_{up}^{-1})_{G_B}(F_A)_j))\,\tilde{\subseteq}\underset{j\in J}{\tilde{\cap}}b(F_A)_j$. We have $a^*((f_{up}^{-1})_{G_B}(F_A)_j)=a((f_{up}^{-1})_{G_B}(F_A)_j)\,\tilde{\cap}G_B\,\tilde{\subseteq}G_B$ and then $f_{up}(\underset{j\in J}{\tilde{\cap}}a^*((f_{up}^{-1})_{G_B}(F_A)_j))\,\tilde{\subseteq}f_{up}(\underset{j\in J}{\tilde{\cap}}G_B)\,\tilde{\subseteq}\underset{j\in J}{\tilde{\cap}}f_{up}(G_B)$. Therefore, $f_{up}(\underset{j\in J}{\tilde{\cap}}a^*((f_{up}^{-1})_{G_B}(F_A)_j))\,\tilde{\subseteq}(\underset{j\in J}{\tilde{\cap}}b(F_A)_j)\,\tilde{\cap}(\underset{j\in J}{\tilde{\cap}}f_{up}(G_B))=\underset{j\in J}{\tilde{\cap}}(b(F_A)_j\,\tilde{\cap}f_{up}(G_B))=$

$\underset{j\in J}{\tilde{\cap}}b^*(F_A)_j$. Thus $\underset{j\in J}{\tilde{\cap}}b^*(F_A)_j\neq\tilde{\phi}$ which proves that $f_{up}(G_B)$ is soft $1-$compact.

Proposition 3.5. Let (X,E_1,a) and (Y,E_2,b) be two fuzzy soft pretopological spaces of type I and $f_{up}:(X,E_1,a)\to(Y,E_2,b)$ be a fuzzy soft bijective pre – continuous function. If G_B is a soft $2-$compact fuzzy soft subset of (X, E_1), then $f_{up}(G_B)$ is a soft $2-$compact subset of (Y, E_2).

Proof. Let $\{(F_A)_j : j \in J\}$, $(F_A)_j \tilde{\subseteq} f_{up}(G_B)$ be a family of fuzzy soft subsets of (Y, E_2) which verifies the $\alpha-$finite intersection property. Note that $(f_{up}^{-1})_{G_B}(F_A)_j$ is the fuzzy soft subset $f_{up}^{-1}(F_A)_j \tilde{\cap} G_B$. For simplisity we use a^* for a_{G_B} and b^* for $b_{f_{up}(G_B)}$. Since $(F_A)_j \tilde{\subseteq} f_{up}(G_B)$, then $f_{up}^{-1}(F_A)_j \tilde{\subseteq} f_{up}^{-1}(f_{up}(G_B)) = G_B$ because f_{up} is fuzzy soft injective and so $\underset{j \in J_o}{\tilde{\cap}} (f_{up}^{-1})_{G_B}(F_A)_j = \underset{j \in J_o}{\tilde{\cap}} f_{up}^{-1}(F_A)_j \tilde{\cap} G_B = \underset{j \in J_o}{\tilde{\cap}} f_{up}^{-1}(F_A)_j$. Therefore, $\psi(\underset{j \in J_o}{\tilde{\cap}} (f_{up}^{-1})_{G_B}(F_A)_j) \geq \alpha$. Thus $(f_{up}^{-1})_{G_B}(F_A)_j$ satisfy the $\alpha-$finite intersection property. Since G_B is soft $2-$compact, then $\psi(\underset{j \in J}{\tilde{\cap}} a^*((f_{up}^{-1})_{G_B}(F_A)_j)) \geq \alpha$. Since f_{up} is fuzzy soft pre – continuous, then

$$
\begin{aligned}
f_{up}(\underset{j \in J}{\tilde{\cap}} a^*((f_{up}^{-1})_{G_B}(F_A)_j)) &\tilde{\subseteq} \underset{j \in J}{\tilde{\cap}} f_{up}(a^*((f_{up}^{-1})_{G_B}(F_A)_j)) \\
&= \underset{j \in J}{\tilde{\cap}} f_{up}(a((f_{up}^{-1})_{G_B}(F_A)_j) \tilde{\cap} G_B) \\
&\tilde{\subseteq} \underset{j \in J}{\tilde{\cap}} (f_{up}(a((f_{up}^{-1})_{G_B}(F_A)_j)) \tilde{\cap} f_{up}(G_B)) \\
&\tilde{\subseteq} \underset{j \in J}{\tilde{\cap}} f_{up}(a((f_{up}^{-1})_{G_B}(F_A)_j)) = \underset{j \in J}{\tilde{\cap}} f_{up}(a(f_{up}^{-1}(F_A)_j \tilde{\cap} G_B)) \\
&= \underset{j \in J}{\tilde{\cap}} f_{up}(a(f_{up}^{-1}(F_A)_j)) \tilde{\subseteq} \underset{j \in J}{\tilde{\cap}} b(f_{up}(f_{up}^{-1}(F_A)_j)) = \underset{j \in J}{\tilde{\cap}} b(F_A)_j
\end{aligned}
$$

Therefore, $f_{up}(\underset{j \in J}{\tilde{\cap}} a^*((f_{up}^{-1})_{G_B}(F_A)_j)) \tilde{\subseteq} \underset{j \in J}{\tilde{\cap}} b(F_A)_j$. We have $a^*((f_{up}^{-1})_{G_B}(F_A)_j) = a((f_{up}^{-1})_{G_B}(F_A)_j) \tilde{\cap} G_B \tilde{\subseteq} G_B$ and then $f_{up}(\underset{j \in J}{\tilde{\cap}} a^*((f_{up}^{-1})_{G_B}(F_A)_j)) \tilde{\subseteq} f_{up}(\underset{j \in J}{\tilde{\cap}} G_B) \tilde{\subseteq} \underset{j \in J}{\tilde{\cap}} f_{up}(G_B)$. Therefore, $f_{up}(\underset{j \in J}{\tilde{\cap}} a^*((f_{up}^{-1})_{G_B}(F_A)_j)) \tilde{\subseteq} (\underset{j \in J}{\tilde{\cap}} b(F_A)_j) \tilde{\cap} (\underset{j \in J}{\tilde{\cap}} f_{up}(G_B)) = \underset{j \in J}{\tilde{\cap}} (b(F_A)_j \tilde{\cap} f_{up}(G_B)) =$

$\tilde{\bigcap}_{j \in J} b^*(F_A)_j$. Thus $\psi(\tilde{\bigcap}_{j \in J} b^*(F_A)_j) \geq \alpha$ which proves that $f_{up}(G_B)$ is soft $2-$compact.

Proposition 3.6. Let $f_{up} : (X, E_1, a) \to (Y, E_2, b)$ be a fuzzy soft surjecive $\text{pre}-$continuous function. If (X, E_1, a) is a soft $1-$compact fuzzy soft pretopological space, then so is (Y, E_2, b).

Proof. Let (X, E_1, a) be a soft $1-$compact fuzzy soft pretopological space and f_{up} be a fuzzy soft surjective $\text{pre}-$continuous function. Let $\{(F_A)_j : j \in J\}$ be a family of fuzzy soft subsets of (Y, E_2) which verifies the finite intersection property. Let $f_{up}^{-1}(F_A)_j = (G_B)_j$, for every $j \in J$. Therefore, $(F_A)_j = f_{up}(G_B)_j$ because f_{up} is fuzzy soft surjective. Since $\tilde{\bigcap}_{j \in J_o} (F_A)_j \neq \tilde{\phi}$, then $f_{up}^{-1}(\tilde{\bigcap}_{j \in J_o} (F_A)_j) \neq \tilde{\phi}$ because f_{up} is fuzzy soft surjective. Therefore, $\tilde{\bigcap}_{j \in J_o} f_{up}^{-1}(F_A)_j \neq \tilde{\phi}$ which implies that $\tilde{\bigcap}_{j \in J_o} (G_B)_j \neq \tilde{\phi}$, but (X, E_1, a) is soft $1-$compact, then $\tilde{\bigcap}_{j \in J} a(G_B)_j \neq \tilde{\phi}$. Since f_{up} is fuzzy soft $\text{pre}-$continuous, then $f_{up}(a(G_B)_j) \tilde{\subseteq} b(f_{up}(G_B)_j)$. Therefore, $\tilde{\bigcap}_{j \in J} f_{up}(a(G_B)_j) \tilde{\subseteq} \tilde{\bigcap}_{j \in J} b(f_{up}(G_B)_j)$.

Now, $\tilde{\bigcap}_{j \in J} b(F_A)_j = \tilde{\bigcap}_{j \in J} b(f_{up}(G_B)_j) \tilde{\supseteq} \tilde{\bigcap}_{j \in J} f_{up}(a(G_B)_j) \tilde{\supseteq} f_{up}(\tilde{\bigcap}_{j \in J} a(G_B)_j) \neq \tilde{\phi}$ which proves that (Y, E_2, b) is soft $1-$compact.

Proposition 3.7. Let $f_{up} : (X, E_1, a) \to (Y, E_2, b)$ be a fuzzy soft surjective $\text{pre}-$continuous function. If (X, E_1, a) is a soft $2-$compact fuzzy soft pretopological space, then so is (Y, E_2, b).

Proof. Let (X, E_1, a) be a soft $2-$compact fuzzy soft pretopological space and f_{up} be a fuzzy soft surjective $\text{pre}-$continuous function. Let $\{(F_A)_j : j \in J\}$ be a family of fuzzy soft subsets of (Y, E_2) which verifies the $\alpha-$finite intersection property. Let $f_{up}^{-1}(F_A)_j = (G_B)_j$, for every $j \in J$. Therefore, $(F_A)_j = f_{up}(G_B)_j$ because

f_{up} is fuzzy soft surjective. Since $\psi(\tilde{\bigcap}_{j\in J_o}(F_A)_j)\geq\alpha$, then $\psi(\tilde{\bigcap}_{j\in J_o} f_{up}^{-1}(F_A)_j)\geq\alpha$. Therefore, $\psi(\tilde{\bigcap}_{j\in J_o}(G_B)_j)\geq\alpha$, but (X,E_1,a) is soft $2-$compact, then $\psi(\tilde{\bigcap}_{j\in J} a(G_B)_j)\geq\alpha$. Since f_{up} is fuzzy soft pre$-$continuous, then $f_{up}(a(G_B)_j)\tilde{\subseteq} b(f_{up}(G_B)_j)$.

Therefore, $\psi(\tilde{\bigcap}_{j\in J} f_{up}(a(G_B)_j))\leq\psi(\tilde{\bigcap}_{j\in J} b(f_{up}(G_B)_j))$.

Now, $\psi(\tilde{\bigcap}_{j\in J} b(F_A)_j)=\psi(\tilde{\bigcap}_{j\in J} b(f_{up}(G_B)_j))\geq\psi(\tilde{\bigcap}_{j\in J} f_{up}(a(G_B)_j))\geq\alpha$ which proves that (Y,E_2,b) is soft $2-$compact.

Definition 3.7. A fuzzy soft pretopological space (X,E,a) of type I is said to be soft $1-$weakly locally compact (soft $2-$weakly locally compact) fuzzy soft pretopological space if for every fuzzy soft point x_e^α in (X,E), there exists $F_A\tilde{\in} FSS(X,E)$ such that $x_e^\alpha\tilde{\in} i_a(F_A)$ and F_A is soft $1-$compact (resp. soft $2-$compact).

Definition 3.8. A fuzzy soft pretopological space (X,E,a) of type I is said to be soft $1-$locally compact (resp. soft $2-$locally compact) fuzzy soft pretopological space if for every fuzzy soft point x_e^α in (X,E) and every $G_B\tilde{\in} FSS(X,E)$ such that $x_e^\alpha\tilde{\in} i_a(G_B)$, there exists $F_A\tilde{\subseteq} G_B$ such that $x_e^\alpha\tilde{\in} i_a(F_A)\tilde{\subseteq} i_a(G_B)$ and F_A is soft $1-$compact (resp. soft $2-$compact).

Lemma 3.1. Let (X,E_1,a) and (Y,E_2,b) be two fuzzy soft pretopological spaces and $f_{up}:(X,E_1,a)\to(Y,E_2,b)$ be a fuzzy soft function. If f_{up} is fuzzy soft pre$-$continuous, then $i_a(f_{up}^{-1}(G_B))\tilde{\supseteq} f_{up}^{-1}(i_b(G_B))$, for every $G_B\tilde{\in} FSS(Y,E_2)$.

Proof. Let f_{up} be fuzzy soft pre$-$continuous and $G_B\tilde{\in} FSS(Y,E_2)$. Then $a(f_{up}^{-1}(G_B^c))\tilde{\subseteq} f_{up}^{-1}(b(G_B^c))$. Taking the complement, we have $(a(f_{up}^{-1}(G_B^c)))^c\tilde{\supseteq}(f_{up}^{-1}(b(G_B^c)))^c$. Thus we have $(a(f_{up}^{-1}(G_B))^c)^c\tilde{\supseteq} f_{up}^{-1}(b(G_B^c))^c$.

By the definition of the fuzzy soft interior function, we have $i_a(f_{up}^{-1}(G_B)) \tilde{\supseteq} f_{up}^{-1}(i_b(G_B))$.

Proposition 3.8. Let (X, E_1, a) and (Y, E_2, b) be two fuzzy soft pretopological spaces of type I. If (X, E_1, a) is soft $1-$locally compact (resp. soft $2-$locally compact) and $f_{up}:(X, E_1, a) \to (Y, E_2, b)$ a fuzzy soft bijective $\text{pre}-$continuous $\text{pre}-$open function, then (Y, E_2, b) is soft $1-$locally compact (resp. soft $2-$locally compact).

Proof. (1) Let y_e^{α} be a fuzzy soft point in (Y, E_2) and $f_{up}^{-1}(y_e^{\alpha})$ its inverse image in (X, E_1). Let $H_C \tilde{\in} FSS(Y, E_2)$ such that $y_e^{\alpha} \tilde{\in} i_b(H_C)$. Assume that $F_A = f_{up}^{-1}(H_C)$. Since f_{up} is fuzzy soft $\text{pre}-$continuous, then $i_a(f_{up}^{-1}(H_C)) \tilde{\supseteq} f_{up}^{-1}(i_b(H_C))$. Since $y_e^{\alpha} \tilde{\in} i_b(H_C)$, then $f_{up}^{-1}(y_e^{\alpha}) \tilde{\in} f_{up}^{-1}(i_b(H_C)) \tilde{\subseteq} i_a(f_{up}^{-1}(H_C)) = i_a(F_A)$. Therefore, $f_{up}^{-1}(y_e^{\alpha}) \tilde{\in} i_a(F_A)$ which implies that there exists $G_B \tilde{\subseteq} F_A$ such that $f_{up}^{-1}(y_e^{\alpha}) \tilde{\in} i_a(G_B) \tilde{\subseteq} i_a(F_A)$ and G_B is soft $1-$compact because (X, E_1, a) is soft $1-$locally compact. Since $G_B \tilde{\subseteq} F_A$, then $G_B \tilde{\subseteq} f_{up}^{-1}(H_C)$. Therefore, $f_{up}(G_B) \tilde{\subseteq} f_{up}(f_{up}^{-1}(H_C)) = H_C$ because f_{up} is fuzzy soft surjective. Thus $f_{up}(G_B) \tilde{\subseteq} H_C$. Since f_{up} is fuzzy soft $\text{pre}-$open, then $f_{up}(i_a(G_B)) \tilde{\subseteq} i_b(f_{up}(G_B))$. Since $f_{up}^{-1}(y_e^{\alpha}) \tilde{\in} i_a(G_B)$, then $f_{up}(f_{up}^{-1}(y_e^{\alpha})) \tilde{\in} f_{up}(i_a(G_B))$. But f_{up} is fuzzy soft surjective, then $y_e^{\alpha} \tilde{\in} f_{up}(i_a(G_B))$.

Therefore, $y_e^{\alpha} \tilde{\in} f_{up}(i_a(G_B)) \tilde{\subseteq} i_b(f_{up}(G_B)) \tilde{\subseteq} i_b(H_C)$. Since f_{up} is fuzzy soft bijective $\text{pre}-$continuous and G_B is soft $1-$compact subset of a fuzzy soft pretopological space of type I, then from Proposition 3.4 we have $f_{up}(G_B)$ is soft $1-$compact. Thus $y_e^{\alpha} \tilde{\in} i_b(f_{up}(G_B)) \tilde{\subseteq} i_b(H_C)$ and $f_{up}(G_B)$ is soft $1-$compact which proves that (Y, E_2, b) is soft $1-$locally compact.

(2) The case of soft $2-$locally compact is similar.

Definition 3.9. Let (X, E_1, a) and (Y, E_2, b) be two fuzzy soft pretopological spaces. A function $f_{up} : (X, E_1, a) \to (Y, E_2, b)$ is said to be a fuzzy soft prehomeomorphism if f_{up} is fuzzy soft bijective, fuzzy soft pre−continuous and fuzzy soft pre−open.

A property which is preserved under fuzzy soft prehomeomorphism is said to be a fuzzy soft pretopological property.

Corollary

Soft 1−compactness, soft 2−compactness, soft 1−locally compactness and soft 2−locally compactness are fuzzy soft pretopological properties.

4. SOFT LINDELÖF AND SOFT COUNTABLE COMPACTNESS

Definition 4.1. A fuzzy soft pretopological space (X, E, a) of type I is called soft 1−lindelöf (resp. soft 2−lindelöf) if for every family $\{(F_A)_j : j \in J\}$ of fuzzy soft subsets of (X, E) with $\underset{j \in J_o}{\tilde{\cap}} (F_A)_j \neq \tilde{\phi}$ (resp. $\psi(\underset{j \in J_o}{\tilde{\cap}} (F_A)_j) \geq \alpha$), for every countable family J_o of J, we have $\underset{j \in J}{\tilde{\cap}} a(F_A)_j \neq \tilde{\phi}$ (resp. $\psi(\underset{j \in J}{\tilde{\cap}} a(F_A)_j) \geq \alpha$).

Theorem 4.1. A fuzzy soft pretopological space (X, E, a) of type I is:-

(1)Soft 1−lindelöf if and only if for every soft a−cover $\{(F_A)_j : j \in J\}$ of (X, E), there exists a countable family J_o of J such that $\{(F_A)_j : j \in J_o\}$ covers (X, E).

(2) Soft 2−lindelöf if and only if for every family $\{(F_A)_j : j \in J\}$ of fuzzy soft subsets of (X, E) such that $\tilde{\psi}(\underset{j \in J}{\tilde{\cup}} i_a (F_A)_j) \not\leq \alpha'$, we can find a countable family J_o of J such that $\tilde{\psi}(\underset{j \in J_o}{\tilde{\cup}} (F_A)_j) \not\leq \alpha'$.

Proof. (1) Similar to the proof of Theorem 3.1.

(2) Similar to the proof of Theorem 3.2.

Definition 4.2. A fuzzy soft pretopological space (X, E, a) of type I is called soft countable $1-$compact (resp. soft countable $2-$compact) if for every countable family $\{(F_A)_j : j \in J_o\}$ of fuzzy soft subsets of (X, E) with $\tilde{\bigcap}_{j \in \Lambda} (F_A)_j \neq \tilde{\phi}$ (resp. $\psi(\tilde{\bigcap}_{j \in \Lambda} (F_A)_j) \geq \alpha$), for every finite subset Λ of J_o, we have $\tilde{\bigcap}_{j \in J_o} a(F_A)_j \neq \tilde{\phi}$ (resp. $\psi(\tilde{\bigcap}_{j \in J_o} a(F_A)_j) \geq \alpha$).

Remark 4.1. (i) Every soft $1-$compact is soft countable $1-$compact and soft $1-$lindelöf.

(ii) Every soft $2-$compact is soft countable $2-$compact and soft $2-$lindelöf.

Theorem 4.2. A fuzzy soft pretopological space (X, E, a) of type I is:-

(1) Soft countable $1-$compact if and only if for every soft countable $a-$cover $\{(F_A)_j : j \in J_o\}$ of (X, E), there exists a finite subset Λ of J_o such that $\{(F_A)_j : j \in \Lambda\}$ covers (X, E).

(2) Soft countable $2-$compact if and only if for every countable family $\{(F_A)_j : j \in J_o\}$ of fuzzy soft subsets of (X, E) such that $\tilde{\psi}(\tilde{\bigcup}_{j \in J_o} i_a(F_A)_j) \not\leq \alpha'$, we can find a finite subset Λ of J_o such that $\tilde{\psi}(\tilde{\bigcup}_{j \in \Lambda} (F_A)_j) \not\leq \alpha'$.

Proof. (1) Similar to the proof of Theorem 3.1.

(2) Similar to the proof of Theorem 3.2.

Proposition 4.1. Every fuzzy soft pre$-$closed subset of a soft $1-$lindelöf (resp. soft $2-$lindelöf) fuzzy soft pretopological space is soft $1-$lindelöf (resp. soft $2-$lindelöf).

Proof. Similar to the proof of Proposition 3.1 (resp. Proposition 3.2).

Proposition 4.2. Let $f_{up}:(X,E_1,a)\to(Y,E_2,b)$ be a fuzzy soft surjective pre−continuous function. If (X,E_1,a) is a soft $1-$lindelöf (resp. soft $2-$lindelöf) fuzzy soft pretopological space, then so is (Y,E_2,b).

Proof. Similar to the proof of Proposition 3.6 (resp. Proposition 3.7).

Theorem 4.3. A fuzzy soft pretopological space (X,E,a) of type IS is:-

(1) Soft $1-$compact if and only if (X,E,a) is soft $1-$lindelöf and soft countable $1-$compact.

(2) Soft $2-$compact if and only if (X,E,a) is soft $2-$lindelöf and soft countable $2-$compact.

Proof. (1) The proof is obvious.

(2) Suppose that (X,E,a) is soft $2-$compact, then (X,E,a) is soft countable $2-$compact and soft $2-$lindelöf from Remark 4.1. Conversely, let (X,E,a) be soft countable $2-$compact and soft $2-$lindelöf and $\{(F_A)_j : j\in J_o\}$ be a countable family of fuzzy soft subsets of (X,E) with $\psi(\tilde{\bigcap}_{j\in\Lambda}(F_A)_j)\geq\alpha$, for every finite subset Λ of J_o. Since (X,E,a) is soft countable $2-$compact, then $\psi(\tilde{\bigcap}_{j\in J_o}a((F_A)_j))\geq\alpha$. Since (X,E,a) is soft $2-$lindelöf, then $\psi(\tilde{\bigcap}_{j\in J}a(a(F_A)_j))\geq\alpha$. But (X,E,a) is of type S which implies that $a(a(F_A)_j)=a(F_A)_j$. Therefore, $\psi(\tilde{\bigcap}_{j\in J}a((F_A)_j))\geq\alpha$. Thus (X,E,a) is soft $2-$compact.

REFERENCES

[1] S. Atmaca and I. Zorlutuna; On fuzzy soft topological spaces, *Annals of Fuzzy Mathematics and Informatics*, 5(2) (2013), 377-386.

[2] R. Badard; Fuzzy pretopological spaces and their representation, *Journal of Mathematical Analysis and Applications*, 81(1981), 378-390.

[3] M. Brissaud; "Les espaces pre'topologiques", *C. R. Acad. Sci. Paris* T-280, 1975.

[4] P. Gain, R. Chakraborty and M. Pal; On compact and semicompact fuzzy soft topological spaces, *J. Math. Comput. Sci*. 4(2014), No. 2, 425-445.

[5] A. Kharal and B. Ahmed; Mappings on Fuzzy Soft Classes, *Advances in Fuzzy Systems*, Article ID 407890, 6 pages, 2009.

[6] F. H. Khedr, M. Azab. Abd-Allah, E. A. Abdelgaber; Fuzzy soft pretopological spaces, *Global Journal of Mathematics*, Vol. 13, No. 2, January 22, 2019, ISSN:2395-4760.

[7] P. K. Maji, R. Biswas and A. R. Roy; Fuzzy soft sets, *J. Fuzzy Math*., 9(3) (2001),

589-602.

[8] A. S. Mashhour, A. A. Ramadan and M. E. Abd EL-monsef; A note on compactness in L −fuzzy pretopological spaces, *Rocky Mountain Journal of Mathematics*, 20(1) (1990), 199-207.

[9]B. Tany and M. B. Kandemir; Topological structure of soft sets, *Computer and Mathematics with Applications*, 61(2011), 2952-2957.

[10] B. P. Varol and H. Aygün, Fuzzy soft toplogy, *Hacettepe Journal of Mathematics and Statistics* 41(3) (2012), 407-419.

الإحكام الناعم في الفضاءات قبل التوبولوجية الفازية الناعمة

فتحي هشام خضر(١)، محمد عزب عبد الله(٢) و إيمان أبو الجود عبد الجابر(٣)

(١)،(٢) قسم الرياضيات - كلية العلوم - جامعة أسيوط

(٣) قسم الرياضيات - كلية العلوم - جامعة المنيا

في هذا البحث تم تقديم ودراسة مفاهيم الإحكام الناعم من النوع الأول والإحكام الناعم من النوع الثاني في الفضاءات قبل التوبولوجية الفازية الناعمة. كذلك تم تقديم ودراسة بعض الأنواع الضعيفة من الإحكام الناعم مثل الإحكام الموضعي الناعم والإحكام قابل للعد الناعم وخاصية ليندلوف الناعم وذلك في هذه الفضاءات.