Investigation of $^{6}\text{Li}+^{24}\text{Mg}$ Elastic Scattering Using Different Folding Models

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In the present study we investigate the $^{6}\text{Li}+^{24}\text{Mg}$ elastic scattering at two energies 88 and 240 MeV in the framework of the optical model. Two optical real potentials are used here, according to $\alpha$-cluster structure of the colliding nuclei. The first double folding (DF) potential for the real central part of the nuclear optical potential is done by folding the $\alpha$-n and $\alpha$-$\alpha$ effective interactions between target and projectile nuclei over the density distributions of $\alpha$-clusters in the target ($^{24}\text{Mg}$) nucleus and considering the $\alpha$-deuteron ($\alpha$-D) structure of the projectile ($^{6}\text{Li}$) nucleus. We call this one is double folding cluster (DFC). The Second potential is DF optical potentials based upon the São Paulo (SP) potential. The imaginary part of the optical potential is calculated in the Woods-Saxon form (WS) for DFC, while for SP both imaginary WS and imaginary folded potentials are used. The experimental angular distributions of the elastic scattering data are successfully obtained using the derived potential. It is found that introducing a real renormalization factor, $N_R$, smaller than unity is essential in order to obtain successful description of the data. The obtained values of $N_R$ in case of SP are more close to unity than those of DFC. The obtained results confirm the validity of the SP to generate nucleus-nucleus optical potentials.

Keywords: Optical model; Elastic scattering; Folding potential; Cluster model; São Paulo potential

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I. INTRODUCTION

The optical model is one of the mostly used models for the description of nuclear scattering especially elastic scattering. The microscopic description of the nucleus–nucleus optical model potential is considered as one of the fundamental tasks in nuclear reaction physics. One of the used methods to
derive the nucleus-nucleus interaction potential is the folding model. The pioneered folding work of Watanabe in his analysis of deuteron projectiles is considered as a review article of this model [1]. In the last few decades, folding model calculations, with microscopic and semi microscopic approaches, were used for the analysis of scattering processes for a large number of interacting systems. Satchler and Love [2,3] have been successfully used M3Y double folding (DF) model for the analysis of light and heavy composite ions scattering. In their analysis, DF optical potential was built on a realistic effective nucleon–nucleon (NN) interaction folded with the nuclear matter density distributions of projectile and target nuclei. The DF model based on matter densities and on the effective NN interaction is successfully used to analyze $\alpha$-nucleus and nucleus-nucleus elastic scattering [4-9]. The elastic scattering of $^6$Li and $^9$Be projectiles were exceptions where the folded potential must be reduced by a renormalization coefficient ($\sim$0.5–0.6) [2, 10-12]. Many studies have been carried out on the scattering of $^6$Li which have distinguished interest. $^6$Li ion falls in the mass range $A = 4 - 12$ of ions whose elastic scattering exhibits a transition [13] between characteristic of light ions ($A\leq4$) and those of heavy ions ($A\geq 12$). On the other hand, it is commonly surmised that, because $^6$Li is weakly bound (1.47 MeV for $^6$Li →$\alpha$+D ), breakup has a large effect on the elastic scattering channel and is responsible for the reduction of the renormalization factor below unity. In our previous study [14] we performed analysis of $^6$Li+$^{28}$Si scattering at low energies (9-20 MeV) by employing the $\alpha$-cluster structure of the colliding nuclei in order to generate semi-microscopic DF potentials considering different targets. In the present work we extend our calculations to analyze $^6$Li+$^{28}$Mg scattering at the two available energies 88 and 240 MeV which are relatively higher energies than those considered previously [14]. In the same time, for the sake of comparison, the considered scattering data are reanalyzed using a microscopic SP potential. So, the present work represents an extension of our previous studies [14-16] in order investigate the validity of $\alpha$-cluster structure of colliding nuclei to construct semi-microscopic folding nucleus-nucleus potentials. The manuscript is organized as follows: in the next section the theoretical formalism is presented, while calculations procedure is given in section III. Results and discussion are demonstrated in section IV and finally conclusions are summarized in section V.

II. FORMALISM

II.1. The semi-microscopic DF cluster model
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Considering the $\alpha$-D cluster structure of $^{6}\text{Li}$ nucleus, the $^{6}\text{Li}+$nucleus semi-microscopic DF potential can be written as [14].

$$V_{6\text{Li}-\text{Target}}^{DFC}(R) = \int |\Psi(Z)|^2$$

$$[V_{\alpha-\text{Target}}(|\vec{R} - \frac{1}{3}\vec{Z}|) + V_{D-\text{Target}}(|\vec{R} + \frac{2}{3}\vec{Z}|)] \ d\vec{Z}$$

(1)

where $R$ is the separation distance between the centers of projectile and target nuclei and $Z$ is the $\alpha$–D separation distance inside $^{6}\text{Li}$ nucleus. $\Psi(Z)$ is the wave function of the relative motion of alpha and D clusters in the ground state of $^{6}\text{Li}$ nucleus. The relative wave function of $^{6}\text{Li}$ can be expressed as [17]

$$\Psi(Z) = \frac{4\beta}{\sqrt{15}} \left(\frac{2\beta}{\pi}\right)^{3/4} Z^2 \exp(-\beta Z^2), \quad \beta = 0.11 \text{ fm}^{-2}$$

(2)

So, considering the $\alpha$-cluster structure of target nucleus, we can formulate the $\alpha$+target single folding potential as

$$V_{\alpha-\text{Target}}(r) = \int \rho_{\text{C Target}}(\vec{r}) V_{\alpha-\vec{r}} \left(|\vec{r} - \vec{r}|\right) d\vec{r} \text{ MeV},$$

(3)

where $\rho_{\text{C Target}}$ is the $\alpha$-cluster distribution density inside the target nucleus. Adopting the composition: $^{24}\text{Mg}\equiv6\alpha$, the $\alpha$-cluster density distribution of the target can be represented in the harmonic oscillator (HO) form as

$$\rho_c(r) = \rho_{0\alpha}(1 + \mu r^2) \exp(-\xi r^2),$$

(4)

with the parameters $\rho_{0\alpha}$, $\mu$ and $\xi$ equal to 0.0502 fm$^{-3}$, 0.6002 fm$^{-2}$ and 0.3173 fm$^{-2}$, respectively. This density has a root mean square (rms) radius of 3.048 fm. The $\alpha$–$\alpha$ interaction, $V_{\alpha-\alpha}$, is represented in the Gaussian form as:

$$V_{\alpha-\alpha}(s) = -V_{0\alpha} \exp(-\kappa s^2) \text{ MeV}.$$  

(5)

Similarly, the $D$–Target interaction potential can be formulated as:

$$V_{D-\text{Target}}(r) = \int \rho_{\text{C Target}}(\vec{r}) V_{D-\vec{r}} \left(|\vec{r} - \vec{r}|\right) d\vec{r} \text{ MeV},$$

(6)

where the $D$–$\alpha$ interaction can be expressed in the form:

$$V_{D-\alpha}(r) = 2 \int \phi(Y) |Y|^2 V_{n-\alpha}(|\vec{r} + \vec{Y}|) d\vec{Y} \text{ MeV},$$

(7)

where $\phi(Y)$ is the wave function of the proton–neutron relative motion inside the deuteron. If we ignore the d-state we can assume the s-state wave function to be [18]

$$\phi(Y) = \left(\frac{2\lambda}{\pi}\right)^{3/4} \exp(-\lambda Y^2), \quad \lambda = 0.053 \text{ fm}^{-2}.$$  

(8)

The $\alpha$–nucleon ($\alpha$–n) interaction, $V_{\alpha-n}$, is taken in a Gaussian form as [19].

$$V_{n-\alpha}(s) = -V_{0n} \exp(-\chi s^2) \text{ MeV}$$

(9)

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Table I. Parameters of the $\alpha-\alpha$ and $n-\alpha$ effective interactions used in Eqs.(5,9)

<table>
<thead>
<tr>
<th>Interaction</th>
<th>$V_{0n}$ ($V_{0\alpha}$) (MeV)</th>
<th>$\chi$ (f m$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha-\alpha$</td>
<td>122.62</td>
<td>0.469</td>
</tr>
<tr>
<td>$n-\alpha$</td>
<td>36.4</td>
<td>0.2657</td>
</tr>
</tbody>
</table>

II.2. The São Paulo potential

Several theoretical potential models have been developed to account for the energy dependence. One of them associates this dependence with nonlocal quantum effects related to the exchange of nucleons between target and projectile [20-22]. It is a global parameter-free optical potential known as the São Paulo optical potential. The SP potential has been successfully used to describe the elastic scattering and peripheral reaction channels for a large number of heavy-ion systems in a very wide energy region, from sub-Coulomb to 200 MeV/nucleon [23-40]. It has been also used to describe the total reaction and fusion cross sections for hundreds of systems [20-22]. Through this model, the bare interaction $V_N$ is correlated to the folding potential $V_F$ as

$$V_N(R, E) = V_F(R) \exp(-4v^2/c^2),$$

(10)

where $c$ is the speed of light and $v$ is the local relative velocity between the two colliding nuclei. So the total nucleus-nucleus potential can be expressed as

$$V^2(R, E) = \frac{2}{\mu} [E - V_c(R) - V_N(R, E)]$$

(11)

where $V_c$ is the Coulomb potential. The velocity-dependence of the potential arises from the effects of the Pauli non-locality [23,41]. The SP potential is obtained numerically by solving Eqs. (10,11) through an iterative process. The folding potential depends on the matter densities of the colliding nuclei as

$$V_F(R) = \int \rho_1(\vec{r}_1)\rho_2(\vec{r}_2)V_0\delta(\vec{R} - \vec{r}_1 + \vec{r}_2)d\vec{r}_1 d\vec{r}_2$$

(12)

with $V_0 = 456$ MeV fm$^3$. The use of the matter densities and delta function in Eq. (12) corresponds to the zero-range approach for the folding potential, which is equivalent [42] to the more usual procedure of using an effective nucleon-nucleon (NN) interaction with the nucleon densities of the nuclei (instead of the matter densities). We considered the two-parameter Fermi (2pF) distribution to describe the nuclear densities

$$\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{\vec{r} - \vec{R}_0}{\alpha}\right)}$$

(13)
The radii of the matter \( (\rho_{OM}) \) and charge \( (\rho_{OC}) \) densities can be well represented using \( R_{OM} = 1.31 A^{1/3} - 0.84 \) fm and \( R_{OC} = 1.76 Z^{1/3} - 0.96 \) fm, respectively. The charge and matter distributions present average diffuseness values of \( \alpha = 0.53 \) fm and \( \alpha = 0.56 \) fm, respectively. The SP potential is based on the systematic of the nuclear densities. The average diffuseness value of this systematic is \( \alpha = 0.56 \) fm. As already commented, small deviations around this average value are expected due to the effects of the structure of the nuclei. In Ref.[42] these variations were analyzed and a standard deviation of \( \Delta = 0.025 \) fm was found. A realistic value for the diffuseness should not be too far from the average value. For the imaginary part of the optical potential, the code can work with two models. It can use a Woods-Saxon (WS) shape or \( W(R) = N_I V(R) \). The WS potential has the 3-parameter form \( W(R) = W_0 f_i(R) \),

\[
f_i(R) = \left[ 1 + \exp\left(\frac{R-R_i}{a_i}\right)\right]^{-1} \cdot R_i = r_i A^{1/3} \tag{14}
\]

The latter model has been applied for several systems in a wide energy range (see Ref.[33]). It was found that the average normalization value \( N_I = 0.8 \) works very well for most of considered scattering systems.

### III. Procedure

To analyze the \(^6\text{Li}+^{24}\text{Mg}\) elastic scattering data at 88 MeV [43] and at 240 MeV [44], we use the DF optical potential generated from the Eqs. (1, 10) considering the following procedure:

1) We neglected the spin-orbit potential since it is well known that elastic scattering cross section data in this energy range are not sensitive to this potential [2].

2) The considered sets of data are analyzed using the DFC expressed by Eq. (1) and four versions of the folded optical SPP (Eq. (10)) denoted as OM1, OM2, OM3 and OM4. The OM1 potential represents only the real part of the interaction (10) and the projectile and target densities taken in 2PF form from SPP while the imaginary part of the optical potential is phenomenologically parameterized in a WS form (14). The OM2 is the same as OM1 but the imaginary part is taken in the folded form (10) normalized by the factor \( N_I \). The OM3 is the same as OM1 but considering the following form [45] of nuclear matter density of \(^6\text{Li}\) instead of the 2pF form.

\[
\rho_p^{\text{Li}}(r_1) = 0.203 \exp(-0.3306 \ r_1^2) + (-0.0131 + 0.001378 \ r_1^2) \exp(-0.1584 \ r_1^2) \text{ fm}^{-3} \tag{15}
\]

The OM4 is the same as OM2 but using the density (15).
3) The obtained potentials are fed into the computer code HIOPTIM-94 [46] to compute the elastic scattering differential cross sections.

4) Routine searches are performed on four free parameters ($N_R$ besides the WS imaginary potential parameters $W$, $r_1$ and $a_1$ for the OM1 and OM3 while two free parameters (the real and imaginary renormalization factors $N_R$ and $N_I$) are searched on for the OM2 and OM4 potentials in order to minimize the chi-squared $\chi^2$, defined as

$$\chi^2 = \frac{1}{N_D} \sum_{k=1}^{N_D} \left[ \frac{\sigma_{th}(\theta_k) - \sigma_{exp}(\theta_k)}{\Delta \sigma_{exp}(\theta_k)} \right]^2$$  \hspace{1cm} (16)

where $\sigma_{th}$ and $\sigma_{exp}$ are the theoretical and the experimental differential cross section, respectively, at the angle $\theta_k$, $N_D$ is the number of angles at which measurements are performed and $\Delta \sigma_{exp}(\theta_k)$ is the error associated with $\sigma_{exp}(\theta_k)$.

IV. RESULTS AND DISCUSSION

The best fit parameters extracted from the auto search using the HIOPTIM-94 code from the derived DFC and SP potentials for the considered elastic scattering data are listed in Tables II and III, respectively. The parameters of the phenomenological WS optical potentials, the corresponding real and imaginary volume integrals per interacting nucleon pair; $J_R$ and $J_I$ and the absorption (reaction) cross sections $\sigma_R$ are also shown. The resulting angular distributions of the elastic scattering differential cross section using the generated real DFC and all types of SP potentials, in conjunction with imaginary WS potentials are shown in Figs.1 and 2 in comparison with the corresponding experimental data.

It is shown that in order to obtain successful reproduction of the data it is necessary to introduce a reducing renormalization factor $N_R$ is far from unity (0.537 for 88 MeV and 0.676 for 240 MeV) for the assumed depths of the considered $\alpha-\alpha$ and $\alpha-n$ effective interactions defined by the parameters listed in Table I. In case of SP potential the renormalization factor $N_R$ is modified and becomes more close to unity for all considered OM types. It is noticed from Figs. 1 and 2 that the extracted predictions of the data using the both derived real semi-microscopic DFC and microscopic SP potentials supplemented by phenomenological imaginary WS potentials (denoted as OM1) are almost identical all over the measured angular ranges. It is noticed from Figs. 1 and 2 and the values of $\chi^2$ shown in Tables II and III that fits with data obtained using DFC, OM1 and OM3 potentials are better than those resulted using OM2 and OM4 potentials. This may be attributed to the more flexibility of the supplemented imaginary phenomenological WS potentials supplemented for
the DFC, OM1 and OM3 potentials based upon three free parameters while in OM2 and OM4 potentials there is only one free parameter considered in the search of folded imaginary potentials. So, the following discussion is restricted to DFC and SP (OM1) only.

Figure 3 presents a comparison between the derived DFC and SP (OM1) renormalized real potentials. It is noticed from the figure that the DFC potentials are apparently deeper than the corresponding SP ones inside the interior region (at small radii). It is observed also from the figure that the consistency between the DFC and SP potentials moves toward the surface region (at larger radii) with increasing the bombarding energy. This indicates that the elastic scattering cross section of the considered reaction is clearly insensitive to the strength (amplitude) of the nuclear potential at small radial distances and the sensitivity is confined to the surface region. The resulting imaginary volume integrals $J_I$ and the absorption (reaction) cross section, $\sigma_R$ deduced for $^6$Li+$^{24}$Mg system using the DFC and SP potentials are listed in Tables II and III.

Finally, regarding the resulted values of the renormalization factor, $N_R$, for the constructed DFC and SP(OM1) potentials listed in Tables II and III, respectively, one may notice that for the DFC potential needs to be normalized with reducing factor which goes down lower than unity. As for the SP potential, it is clearly noted that $N_R$ value is better than those of DFC potential.

So, from the investigation of the present results and those of our previous study one may extract a confirmation that the generated DFC potential is able without renormalization to successfully reproduce $^6$Li elastic scattering data at relatively low bombarding energies. At higher energies, DFC potential needs a renormalization of about $0.6 \pm 0.05$. On the other side, the present results provide an additional evidence for the ability of the SPP to successfully describe $^6$Li elastic scattering with renormalization close to unity and similar to work of [44]. In Ref. [44], the authors used density dependent version of M3Y interaction called CDM3Y6. However, in the same time, the present results indicate that the constructed standard SP potential [20-22] needs to be renormalized by a reducing factor in order to produce successful predictions of the data at higher energies.

V. SUMMARY AND CONCLUSIONS

The present work is directed to analyze $^6$Li+$^{24}$Mg elastic scattering in the framework of DF optical model based upon the $\alpha$-cluster. The derived DF potentials are considered as the real part of the nuclear optical potentials. The imaginary part is treated phenomenologically through the WS form in order to analyze the elastic scattering differential cross section of experimental data in
the considered energy range. Successful description of the data is obtained with the derived semi-microscopic DFC real potentials.

For the sake of comparison, the same elastic scattering data are reanalyzed using microscopic SP potentials. It is found that although the DFC and SP potentials are generated in the framework of different folding approaches based upon different ingredients (matter densities and effective interactions) they produce similar predictions of the data all over the measured angular ranges. So, the results of the present work provide an additional evidence that the α-cluster structure can be successfully used to construct semi-microscopic nuclear potentials for light heavy nuclei. While SP potential is a good candidate for describing the data with better results than those of CDM3Y6 and DFC potentials[47,48].

Finally, it is worthwhile to point out that our previous and present analysis reveal a considerable confirmation of the substantial ability of the DFC potentials to reproduce elastic scattering data for light heavy nuclei. The cluster model was successfully used to describe many reactions such as α +40Ca [49]. Therefore, it is greatly recommended to perform more investigations on the analysis of elastic and inelastic scattering and other nonelastic channels using the DFC potential. The extension of this work to other energies and targets is important to complete the picture. Recently, elastic scattering of 6Li+28Si at seven energies in the energy range from 76 to 318 MeV is prepared to be published. While, elastic scattering of 6Li+40Ca at nine energies in the energy range from 26 to 240 MeV is submitted to [50]. In the same time SP is

Table II. Best fitting parameters obtained for 6Li+24Mg elastic scattering using the DFC potential.

<table>
<thead>
<tr>
<th>$E_{Lab}$ MeV</th>
<th>$N_R$</th>
<th>$W_0$ MeV</th>
<th>$R_l$ fm</th>
<th>$a_l$ fm</th>
<th>$J_R$ MeVfm$^3$</th>
<th>$J_I$ MeVfm$^3$</th>
<th>$\sigma_R$ mb</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>88</td>
<td>0.537</td>
<td>8.291</td>
<td>1.5636</td>
<td>0.7929</td>
<td>214.16</td>
<td>117.86</td>
<td>1664</td>
<td>11.40</td>
</tr>
<tr>
<td>240</td>
<td>0.676</td>
<td>34.523</td>
<td>0.9317</td>
<td>1.2276</td>
<td>269.84</td>
<td>149.98</td>
<td>1962</td>
<td>9.05</td>
</tr>
</tbody>
</table>
Table III. Same as Table III but using the SP model.

<table>
<thead>
<tr>
<th>$E_{lab}$ MeV</th>
<th>Potential Shape</th>
<th>$W_0$, MeV</th>
<th>$N_R$</th>
<th>$W_1$, MeV</th>
<th>( r_1 ), fm</th>
<th>$a_0$, fm</th>
<th>$J_R$, MeVfm$^3$</th>
<th>$J_I$, MeVfm$^3$</th>
<th>$\sigma_R$, mb</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>88</td>
<td>OM1</td>
<td>27.14</td>
<td>0.785</td>
<td>1.546</td>
<td>1.1904</td>
<td>1.0206</td>
<td>275.89</td>
<td>149.81</td>
<td>1899</td>
<td>19.74</td>
</tr>
<tr>
<td></td>
<td>OM2</td>
<td>240</td>
<td>0.697</td>
<td>1.258</td>
<td>-</td>
<td>-</td>
<td>244.80</td>
<td>353.59</td>
<td>1567</td>
<td>82.88</td>
</tr>
<tr>
<td></td>
<td>OM3</td>
<td>2.560</td>
<td>0.699</td>
<td>156.88</td>
<td>1.0693</td>
<td>0.9658</td>
<td>258.48</td>
<td>275.28</td>
<td>1800</td>
<td>21.96</td>
</tr>
<tr>
<td></td>
<td>OM4</td>
<td>0.642</td>
<td>1.089</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>237.16</td>
<td>332.38</td>
<td>1618</td>
<td>34.51</td>
</tr>
<tr>
<td>240</td>
<td>OM1</td>
<td>0.831</td>
<td>34.518</td>
<td>0.9599</td>
<td>1.0219</td>
<td>-</td>
<td>240.89</td>
<td>139.05</td>
<td>1664</td>
<td>6.33</td>
</tr>
<tr>
<td></td>
<td>OM2</td>
<td>0.996</td>
<td>1.459</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>288.82</td>
<td>338.41</td>
<td>1464</td>
<td>16.12</td>
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<tr>
<td></td>
<td>OM3</td>
<td>0.935</td>
<td>53.482</td>
<td>0.9089</td>
<td>0.9171</td>
<td>-</td>
<td>285.25</td>
<td>176.63</td>
<td>1591</td>
<td>4.50</td>
</tr>
<tr>
<td></td>
<td>OM4</td>
<td>0.984</td>
<td>1.307</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>300.17</td>
<td>318.96</td>
<td>1509</td>
<td>13.01</td>
</tr>
</tbody>
</table>

Fig. 1 $^6$Li+$^{24}$Mg elastic scattering using the DFC potential extracted from expressions (1) in comparison with experimental data.
Fig. 2  \( ^6\text{Li}+^{24}\text{Mg} \) elastic scattering using the SP potential extracted from expressions (10) in comparison with experimental data.
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**Fig. 3:** A comparison between the derived real normalized DFC and SP potentials.

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**Reference**


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دراسة التشتت المرئي للنواة الليثيوم $^6\text{Li}$ والواغنيسيوم $^{24}\text{Mg}$ باستخدام صور متنوعة

للنموذج الطي

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قسم الفيزياء جامعة اسوان

قسم الفيزياء جامعة الازهر فرع اسيوط

في هذه الدراسة تقوم بدراسة التشتت المرئي لتفاعل الاتي

$^6\text{Li}^{+24}\text{Mg}$

للنواة المتصادمة $\alpha - \text{cluster}$ عند الطاقة $880$ مليون الكترون وولت في إطار النموذج البصري. باستخدام نموذجين مختلفين وهما DFC , Sao_paulo نموذج الطي المزدوج يفتقر بواسطة الطي على التفاعل الفعال $n - \alpha$ و $\alpha - \alpha$ و $\alpha - \text{cluster}$ لنموذج الطي المزدوج يفتقر بواسطة الطي على التفاعل الفعال بين نواة الفا ونواة الليثيوم. ويعتمد الجزء الحقيقي للنموذج على الترتيب العكسي $\alpha - \text{cluster}$ في النموذج عناصر نواة الليثيوم وحذا الفا والديزون وسمي هذا النموذج بنموذج الطي المزدوج العكسي. والنموذج الثاني هو عبارة عن نموذج الطي المزدوج البصري ويعتمد ناتج الجزء التخيلي لجهد البصري في صورة الودزو ساكسون Sao_Paulo. على بالنسبة لنموذج الطي المزدوج العكسي، بالنسبة ساوكولو في صورة الودزو ساكسون وفي صورة الجهد الطي التخيلي. وكلا النموذجين أعطي توافق مع الداتا العملية المقاسة ولاحتظنا أن قيمة $N_R$ تقترب من الواحد الصحيح.

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