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A Transmuted Survival Kappa Distribution: Properties and Applications

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ABSTRACT

The author proposes a new four-parameter distribution that aims to provide a more flexible approach to modelling real-life data compared to commonly used lifetime distributions defined. We define a new lifetime model called (Transmuted Survival Kappa Distribution (TSK)), is derived using a transmuted survival method. The methodology involves using “The Probability Density function and cumulative distribution Function” of the Kappa distribution as the foundation of TSK. By substituting these functions into the transmuted survival model, the authors create a novel and adaptable lifetime distribution capable of accurately modelling real-life data.

The study’s conclusions highlight that the hazard rate of TSK increases over time. Furthermore, the authors demonstrate that TSK offers a significantly better fit compared to existing distributions like: the three-parameter Kappa distribution, three-parameter Lindely distribution (LTD), exponential distribution (EXP) and Weibull distribution (WD). Notably, new distribution has the ability to model hazard rates, this makes it particularly suitable for data that other distributions do not fit. Therefore, we suggest using the new distribution for applications in various fields, including, reliability studies, earthquake, data analysis, and more.

1. INTRODUCTION

Statistical researchers encounter numerous challenges when analyzing data and estimating distribution-related parameters. One crucial problem they face is selecting an appropriate distribution for data. Researchers in the statistical field have been enhancing probability distributions, transforming them to achieve the most accurate representation of data with minimal errors. This transformation becomes especially vital when researchers encounter issues in choosing samples with equal probabilities. In such cases, the original distribution proves inadequate for modelling phenomena a new Extended Lindley distribution by using the Weibull link function introduced by [1]. A new survival model is adaptable for produce a new lifetime distribution for analyzing positive data, called the transmuted survival exponential distribution (TSE)”, proposed by [2]. A new generalization of the transmuted Lindley distribution introduced by [3]. The transmuted Zeghdoudi distribution is derived by using the quadratic rank transmutation map approach presented by [4]. The transmuted logistic-exponential distribution with three parameters with the aim of increasing the shape flexibility of logistic-exponential (LE) distribution proposed by [5], study new distribution called transmuted Weibull distribution. The usefulness of the new distribution for modelling data is illustrated using real data set presented by [6]. A transmuted modified Weibull distribution contains eleven life time distributions as special cases presented by [7]. By using the quadratic rank transmutation map (QRTM) in order to generate a flexible family of probability distribution considering Fréchet distribution as the base distribution proposed by [8]. The transformed power function distribution (TPF) by applied to three real datasets to demonstrate its usefulness [9] and A new distribution of five parameters a called transmuted Weibull Lomax distribution suggested by [10].

2. METHODOLGY

In this section, let the random variable K follow the Kappa Distribution with parameters (α, β, θ). Then probability density function (PDF), cumulative distribution function (CDF) and survival function can be expressed as follows

$$f^*(k) = \begin{cases} \dfrac{\alpha\theta}{\beta}\left(\dfrac{k}{\beta}\right)^{\theta-1}\left[\alpha + \left(\dfrac{k}{\beta}\right)^{\alpha\theta}\right]^{-\frac{\alpha+1}{\alpha}}, & k > 0 \ (\beta, \alpha, \theta > 0) \\ 0, & \text{otherwise} \end{cases} ; \qquad (1)$$

$$F^*(k) = \begin{cases} \left[\dfrac{\left(\frac{k}{\beta}\right)^{\alpha\theta}}{\alpha + \left(\frac{k}{\beta}\right)^{\alpha\theta}}\right]^{\frac{1}{\alpha}}, & k > 0 \ (\beta, \alpha, \theta > 0) \\ 0, & \text{otherwise} \end{cases} \qquad (2)$$

$$S^*(k) = 1 - F^*(k) = 1 - \left[\frac{\left(\frac{k}{\beta}\right)^{\alpha\theta}}{\alpha + \left(\frac{k}{\beta}\right)^{\alpha\theta}}\right]^{\frac{1}{\alpha}}. \qquad (3)$$

The purpose of this paper is to find another generalization for Kappa distribution called the transmuted survival Kappa distribution (TSK), we can calculate a new S(k) from the following converted formula [2]:

$$S(k)_{TSK} = (1+\gamma)[S^*(k)]^2 - \gamma S^*(k) \quad (4)$$

where: $S^*(k)$ is the survival function of the original distribution.

γ: is a real number called transmuted parameter ($\gamma \geq 0$).

By differentiation law, we have

$$dS(k)_{TSK} = -dF(k) = -f(k)$$

Then from Eq.(4), we have

$$dS(k)_{TSK} = 2(1+\gamma)[S^*(k)] \;\; dS^*(k) - \gamma dS^*(k)$$

From Eq.4 in Eq.3, we get

$$-f(k) = -2(1+\gamma)S^*(k)f^*(k) + \gamma f^*(k)$$

$$f(k) = 2(1+\gamma)S^*(k)f^*(k) - \gamma f^*(k)$$

then

$$f(k) = f^*(k)[2(1+\gamma)S^*(k) - \gamma]. \quad (5)$$

At (k = 0) then a function f(0) is defined f: R→$[0, \infty]$ is a pdf if only if :

1) $f(k) \geq 0$, for all $(k > 0)$.

2) $\int_0^\infty f(k)dk = 1$

From Eq.5 at $f(k) > f^*(k)$, we get

$$f(k) = f^*(k)[2(1+\gamma)S(k)^* - \gamma]$$

$$0 < [2(1+\gamma)S^*(k) - \gamma]$$

$$0 < [2S^*(k) - \gamma(1 - 2S^*(k))]$$

Then

$$\gamma < \frac{2S^*(k)}{1-2S^*(k)}$$

and we find

$$\int_0^\infty f(k)dk = 2(1+\gamma)\int_0^\infty S^*(k)f^*(k)dk - \int_0^\infty \gamma f^*(k)dk$$

let $u = F^{*}(k), S^{*}(k) = 1 - u$, $du = dF^{*}(k) = f^{*}(k)dk$

Then, we have

$$2(1+\gamma)\int_0^1 (1-u)du - \gamma\int_0^1 du = (2+2\gamma)\left(u - \frac{u^2}{2}\right)\Bigg]_0^1 - \gamma u\Big]_0^1 \qquad (6)$$

$= 1 + \gamma - \gamma$

So that $\int_0^{\infty} f(k)dk = 1$.

2.1 The Transmuted Survival Kappa Distribution

The survival function of the TSK is obtained by substituting in Eq.4, from Eq. 3,

$$S_{TSK} = (1+\gamma)\left[1 - \left[\frac{\left(\frac{k}{\beta}\right)^{\alpha\theta}}{\alpha + \left(\frac{k}{\beta}\right)^{\alpha\theta}}\right]^{\left(\frac{1}{\alpha}\right)}\right]^2 - \gamma\left[1 - \left[\frac{\left(\frac{k}{\beta}\right)^{\alpha\theta}}{\alpha + \left(\frac{k}{\beta}\right)^{\alpha\theta}}\right]^{\frac{1}{\alpha}}\right] \qquad (7)$$

The graph for the TSK can be seen in Figure 1

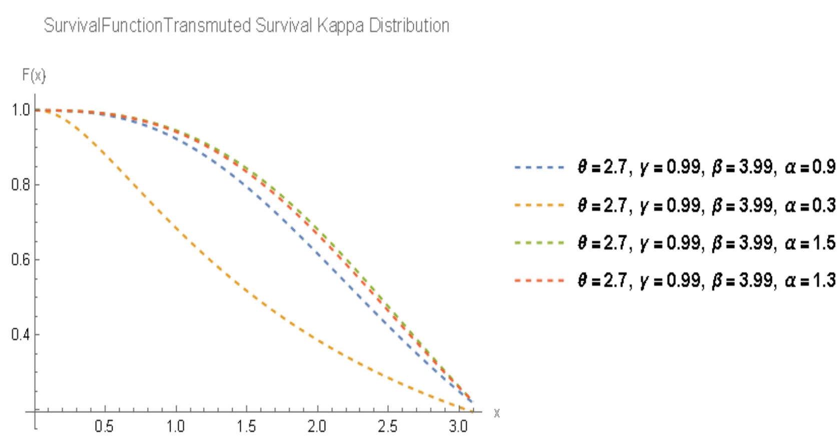

Figure 1: The Survival Function $(S)_{TSK}$

The CDF of the TSK is given by

$$F(k)_{TSK} = 1 - S(k)_{TSK}$$

$$F(k)_{TSK} = 1 - \left((1+\gamma)\left[1 - \left[\frac{\left(\frac{k}{\beta}\right)^{\alpha\theta}}{\alpha + \left(\frac{k}{\beta}\right)^{\alpha\theta}}\right]^{\left(\frac{1}{\alpha}\right)}\right]^2 - \gamma\left[1 - \left[\frac{\left(\frac{k}{\beta}\right)^{\alpha\theta}}{\alpha + \left(\frac{k}{\beta}\right)^{\alpha\theta}}\right]^{\frac{1}{\alpha}}\right]\right) \quad (8)$$

The graph for the TSK can be seen in Figure 2:

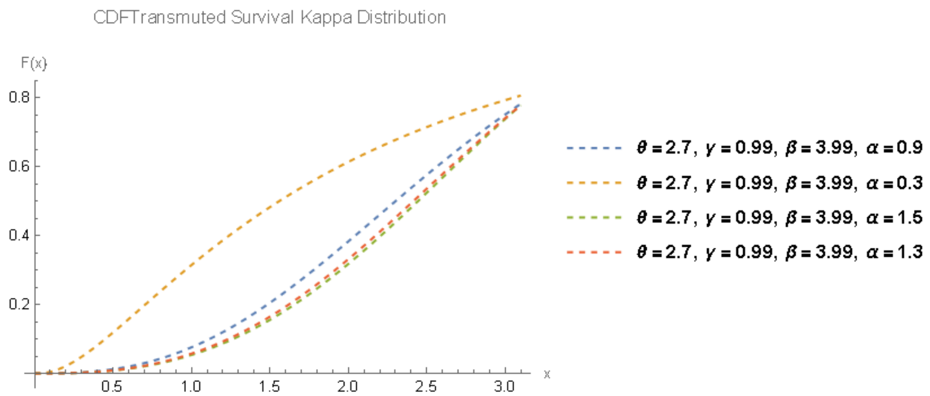

Figure 2: The (pdf) of TSK

From Eq. (5) we can get pdf for new distribution

$$f(k)_{TSK} = \frac{\alpha\theta}{\beta}\left(\frac{k}{\beta}\right)^{\theta-1}\left[\alpha + \left(\frac{k}{\beta}\right)^{\alpha\theta}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\left[2(1+\gamma)1 - \left[\frac{\left(\frac{k}{\beta}\right)^{\alpha\theta}}{\alpha + \left(\frac{k}{\beta}\right)^{\alpha\theta}}\right]^{\left(\frac{1}{\alpha}\right)} - \gamma\right] \quad (9)$$

The graph for the TSK can be seen in Figure 3**:**

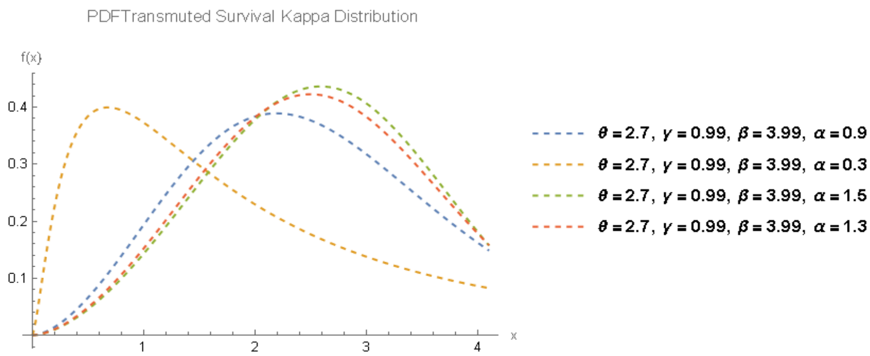

Figure 3: The pdf for TSK.

By definition, the hazard function of a random variable k defind as:

$$h(k,\beta,\alpha,\theta,\gamma)_{TSK}=\frac{f(k)_{TSK}}{1-F(k)_{TSK}}$$

And the hazard function for new (TSK) distribution is given as:

$$h(k)_{TSk}=\frac{\frac{\alpha\theta}{\beta}\left(\frac{k}{\beta}\right)^{\theta-1}\left(\alpha+\left(\frac{k}{\beta}\right)^{\alpha\theta}\right)^{-\left(\frac{\alpha+1}{\alpha}\right)}\left[2(1+\gamma)1-\left(\frac{\left(\frac{k}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{t}{\beta}\right)^{\alpha\theta}}\right)^{\left(\frac{1}{\alpha}\right)}-\gamma\right]}{(1+\gamma)\left[1-\left(\frac{\left(\frac{k}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{k}{\beta}\right)^{\alpha\theta}}\right)^{\left(\frac{1}{\alpha}\right)}\right]^{2}-\gamma\left[1-\left(\frac{\left(\frac{k}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{k}{\beta}\right)^{\alpha\theta}}\right)^{\frac{1}{\alpha}}\right]} \qquad (10)$$

3. STATISTICAL CHARACTERISTICS

In this section, we will find some characteristics of the new distribution (TSK), the main ones are:

3.1 Quintile Function

This function is equal to the inverse Cumulative Distribution Function according to the following formula:

$\text{let}:\ k=Q(u),$

$$Q(u) = F^{-1}(u),$$

Then: $k = F^{-1}(u), \quad 0 < u < 1$

$$u = \left(1 - (1 + \gamma)\left[1 - \left[\frac{\left(\frac{k}{\beta}\right)^{\alpha\theta}}{\alpha + \left(\frac{k}{\beta}\right)^{\alpha\theta}}\right]^{\frac{2}{\alpha}}\right] - \gamma\left[1 - \left[\frac{\left(\frac{k}{\beta}\right)^{\alpha\theta}}{\alpha + \left(\frac{k}{\beta}\right)^{\alpha\theta}}\right]^{\frac{1}{\alpha}}\right]\right)^{-1}$$

$$k = \beta\left(\alpha\left(-1 + 2^{\alpha}\left(-\frac{\gamma + \sqrt{-4u(1+\gamma) + (2+\gamma)^2}}{1+\gamma}\right)^{-\alpha}\right)\right)^{\frac{1}{\alpha\theta}} \qquad (11)$$

3.2. Moments

Let $X \sim TSK\,(x, \beta, \alpha, \theta, \gamma)$ then r^{th} order moment about origin of μ_r is:

$$E(X^r) = \mu'_r = \int_{-\infty}^{\infty} x^r f(x; \beta, \alpha, \theta, \gamma)_{TSK}.\, dx$$

$$E(X^r) = \mu'_r = \int_0^{\infty} x^r \frac{\alpha\theta}{\beta}\left(\frac{x}{\beta}\right)^{\theta-1}\left[\alpha + \left(\frac{x}{\beta}\right)^{\alpha\theta}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\left[2(1+\gamma)\left(1 - \left[\frac{\left(\frac{x}{\beta}\right)^{\alpha\theta}}{\alpha + \left(\frac{x}{\beta}\right)^{\alpha\theta}}\right]^{\left(\frac{1}{\alpha}\right)}\right) - \gamma\right].dx \qquad (12)$$

$$E(X^r) = \mu'_r = \left(\frac{2\beta^{r+1}}{\theta}\alpha^{\frac{r-2\alpha\theta}{\alpha\theta}}\left[\frac{\Gamma\frac{2\theta + r}{\alpha\theta}\Gamma\frac{\alpha\theta - r}{\alpha\theta}}{\Gamma\frac{2+\alpha}{\alpha}}\right]\right) + \left(\frac{2\gamma\beta^{r+1}}{\theta}\alpha^{\frac{r-2\alpha\theta}{\alpha\theta}}\left[\frac{\Gamma\frac{2\theta + r}{\alpha\theta}\Gamma\frac{\alpha\theta - r}{\alpha\theta}}{\Gamma\frac{2+\alpha}{\alpha}}\right]\right) + \gamma\beta^r\alpha^{\frac{r}{\alpha\theta}-1}\left[\frac{\Gamma\frac{r+\theta}{\alpha\theta}\Gamma 1 - \frac{r}{\alpha\theta}}{\Gamma\frac{\alpha+1}{\alpha}}\right]$$

When r=1, $E(X)$ is

$$E(X) = \mu'_1 = \left[\left(\frac{2\beta^2}{\theta}\alpha^{\frac{1-2\alpha\theta}{\alpha\theta}}\left[\frac{\Gamma\frac{2\theta+1}{\alpha\theta}\Gamma\frac{\alpha\theta-1}{\alpha\theta}}{\Gamma\frac{2+\alpha}{\alpha}}\right]\right)(1+\gamma)\right] + \gamma\beta^1\alpha^{\frac{1}{\alpha\theta}-1}\left[\frac{\Gamma\frac{1+\theta}{\alpha\theta}\Gamma 1-\frac{1}{\alpha\theta}}{\Gamma\frac{\alpha+1}{\alpha}}\right]$$

when r=2, $E(X^2)$ is:

$$E(X^2) = \mu'_2 = \left(\frac{2\beta^3}{\theta}\alpha^{\frac{2-2\alpha\theta}{\alpha\theta}}\left[\frac{\Gamma\frac{2\theta+2}{\alpha\theta}\Gamma\frac{\alpha\theta-2}{\alpha\theta}}{\Gamma\frac{2+\alpha}{\alpha}}\right]\right)(1+\gamma) + \gamma\beta^2\alpha^{\frac{2}{\alpha\theta}-1}\left[\frac{\Gamma\frac{2+\theta}{\alpha\theta}\Gamma 1-\frac{2}{\alpha\theta}}{\Gamma\frac{\alpha+1}{\alpha}}\right]$$

When r=3, $E(X^3)$ is:

$$E(X^3) = \mu'_3 = \left(\frac{2\beta^4}{\theta}\alpha^{\frac{3-2\alpha\theta}{\alpha\theta}}\left[\frac{\Gamma\frac{2\theta+3}{\alpha\theta}\Gamma\frac{\alpha\theta-3}{\alpha\theta}}{\Gamma\frac{2+\alpha}{\alpha}}\right]\right)(1+\gamma) + \gamma\beta^3\alpha^{\frac{3}{\alpha\theta}-1}\left[\frac{\Gamma\frac{3+\theta}{\alpha\theta}\Gamma 1-\frac{3}{\alpha\theta}}{\Gamma\frac{\alpha+1}{\alpha}}\right]$$

When r=4, $E(X^4)$ is:

$$E(X^4) = \mu'_4 = \left(\frac{2\beta^5}{\theta}\alpha^{\frac{4-2\alpha\theta}{\alpha\theta}}\left[\frac{\Gamma\frac{2\theta+4}{\alpha\theta}\Gamma\frac{\alpha\theta-4}{\alpha\theta}}{\Gamma\frac{2+\alpha}{\alpha}}\right]\right)(1+\gamma) + \gamma\beta^4\alpha^{\frac{4}{\alpha\theta}-1}\left[\frac{\Gamma\frac{4+\theta}{\alpha\theta}\Gamma 1-\frac{4}{\alpha\theta}}{\Gamma\frac{\alpha+1}{\alpha}}\right]$$

3.3. Moments about The Mean

Let X denote the random variable follows TSKD, then moments about the mean order moment about origin of μ_r is:

$$E(X-\mu)^r = \int_0^\infty (x-\mu)^r \; f(x).dx \tag{13}$$

$$E(X-\mu)^r = \left\{ \begin{array}{l} \beta^j \alpha^{\frac{j}{\alpha\theta}-1} \sum_{j=0}^{r} \binom{r}{j} (-\mu)^{r-j} \beta^j \alpha^{\frac{j}{\alpha\theta}-1} \left[\frac{\Gamma \frac{j}{\alpha\theta} + \frac{1}{\alpha} \Gamma 1 - \frac{j}{\alpha\theta}}{\Gamma 1 + \frac{1}{\alpha}} \right] (2+\gamma) \\ -\frac{2\alpha\theta(1+\gamma)}{\beta} \frac{\beta^{r+1}}{\alpha\theta} \alpha^{\frac{r-\theta}{\alpha\theta}} \sum_{j=0}^{r} \binom{r}{j} \beta^j \alpha^{\frac{j}{\alpha\theta}-1} (-\mu)^{r-j} \left[\frac{\Gamma \frac{2\theta+j}{\alpha\theta}, \Gamma \frac{\alpha\theta+j}{\alpha\theta}}{\Gamma \frac{2\theta+2j+\alpha\theta}{\alpha\theta}} \right] \end{array} \right\} \quad (14)$$

Now, we obtain the first four moments of the TSK distribution by putting r = 2,3,4... n, in Eq.(14) as:

When r=2

$$E(X-\mu)^2 = \left\{ \begin{array}{l} \beta^j \alpha^{\frac{j}{\alpha\theta}-1} \sum_{j=0}^{2} \binom{2}{j} (-\mu)^{2-j} \beta^j \alpha^{\frac{j}{\alpha\theta}-1} \left[\frac{\Gamma \frac{j}{\alpha\theta} + \frac{1}{\alpha} \Gamma 1 - \frac{j}{\alpha\theta}}{\Gamma 1 + \frac{1}{\alpha}} \right] (2+\gamma) \\ -\frac{2\alpha\theta(1+\gamma)}{\beta} \frac{\beta^{3}}{\alpha\theta} \alpha^{\frac{2-\theta}{\alpha\theta}} \sum_{j=0}^{2} \binom{r}{j} \beta^j \alpha^{\frac{j}{\alpha\theta}-1} (-\mu)^{2-j} \left[\frac{\Gamma \frac{2\theta+j}{\alpha\theta}, \Gamma \frac{\alpha\theta+j}{\alpha\theta}}{\Gamma \frac{2\theta+2j+\alpha\theta}{\alpha\theta}} \right] \end{array} \right\}$$

$$\sigma^2 = \left\{ \begin{array}{l} \beta^j \alpha^{\frac{j}{\alpha\theta}-1} \sum_{j=0}^{2} \binom{2}{j} (-\mu)^{2-j} \beta^j \alpha^{\frac{j}{\alpha\theta}-1} \left[\frac{\Gamma \frac{j}{\alpha\theta} + \frac{1}{\alpha} \Gamma 1 - \frac{j}{\alpha\theta}}{\Gamma 1 + \frac{1}{\alpha}} \right] (2+\gamma) \\ -\frac{2\alpha\theta(1+\gamma)}{\beta} \frac{\beta^{3}}{\alpha\theta} \alpha^{\frac{2-\theta}{\alpha\theta}} \sum_{j=0}^{2} \binom{2}{j} \beta^j \alpha^{\frac{j}{\alpha\theta}-1} (-\mu)^{2-j} \left[\frac{\Gamma \frac{2\theta+j}{\alpha\theta}, \Gamma \frac{\alpha\theta+j}{\alpha\theta}}{\Gamma \frac{2\theta+2j+\alpha\theta}{\alpha\theta}} \right] \end{array} \right\} \quad (15)$$

$$\sigma = \sqrt{\left\{ \begin{array}{l} \beta^j \alpha^{\frac{j}{\alpha\theta}-1} \sum_{j=0}^{2} \binom{2}{j} (-\mu)^{2-j} \beta^j \alpha^{\frac{j}{\alpha\theta}-1} \left[\frac{\Gamma \frac{j}{\alpha\theta} + \frac{1}{\alpha} \Gamma 1 - \frac{j}{\alpha\theta}}{\Gamma 1 + \frac{1}{\alpha}} \right] (2+\gamma) \\ -\frac{2\alpha\theta(1+\gamma)}{\beta} \frac{\beta^{3}}{\alpha\theta} \alpha^{\frac{2-\theta}{\alpha\theta}} \sum_{j=0}^{2} \binom{2}{j} \beta^j \alpha^{\frac{j}{\alpha\theta}-1} (-\mu)^{2-j} \left[\frac{\Gamma \frac{2\theta+j}{\alpha\theta}, \Gamma \frac{\alpha\theta+j}{\alpha\theta}}{\Gamma \frac{2\theta+2j+\alpha\theta}{\alpha\theta}} \right] \end{array} \right\}} \quad (16)$$

When r=3

$$E(X-\mu)^3=\left\{\begin{array}{l}\beta^j\alpha^{\frac{j}{\alpha\theta}-1}\sum_{j=0}^{3}\binom{3}{j}(-\mu)^{3-j}\beta^j\alpha^{\frac{j}{\alpha\theta}-1}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{1}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}}{\Gamma 1+\frac{1}{\alpha}}\right](2+\gamma)\\ -\frac{2\alpha\theta(1+\gamma)}{\beta}\frac{\beta^4}{\alpha\theta}\alpha^{\frac{r-\theta}{\alpha\theta}}\sum_{j=0}^{3}\binom{3}{j}\beta^j\alpha^{\frac{j}{\alpha\theta}-1}(-\mu)^{3-j}\left[\frac{\Gamma\frac{2\theta+j}{\alpha\theta},\Gamma\frac{\alpha\theta+j}{\alpha\theta}}{\Gamma\frac{2\theta+2j+\alpha\theta}{\alpha\theta}}\right]\end{array}\right\}$$

When r=4

$$E(X-\mu)^4=\left\{\begin{array}{l}\beta^j\alpha^{\frac{j}{\alpha\theta}-1}\sum_{j=0}^{4}\binom{4}{j}(-\mu)^{4-j}\beta^j\alpha^{\frac{j}{\alpha\theta}-1}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{1}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}}{\Gamma 1+\frac{1}{\alpha}}\right](2+\gamma)\\ -\frac{2\alpha\theta(1+\gamma)}{\beta}\frac{\beta^5}{\alpha\theta}\alpha^{\frac{4-\theta}{\alpha\theta}}\sum_{j=0}^{4}\binom{4}{j}\beta^j\alpha^{\frac{j}{\alpha\theta}-1}(-\mu)^{4-j}\left[\frac{\Gamma\frac{2\theta+j}{\alpha\theta},\Gamma\frac{\alpha\theta+j}{\alpha\theta}}{\Gamma\frac{2\theta+2j+\alpha\theta}{\alpha\theta}}\right]\end{array}\right\}$$

3.4 Coefficient of Variation

Mathematically, the formula for calculating the Coefficient of Variation (CV) is:

$$(CV)_{TSK}=\frac{\sigma}{\mu}\times 100\%$$

$$(CV)_{TSK}=\frac{\sqrt{\left\{\begin{array}{l}\beta^j\alpha^{\frac{j}{\alpha\theta}-1}\sum_{j=0}^{2}\binom{2}{j}(-\mu)^{2-j}\beta^j\alpha^{\frac{j}{\alpha\theta}-1}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{1}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}}{\Gamma 1+\frac{1}{\alpha}}\right](2+\gamma)\\ -\frac{2\alpha\theta(1+\gamma)}{\beta}\frac{\beta^3}{\alpha\theta}\alpha^{\frac{2-\theta}{\alpha\theta}}\sum_{j=0}^{2}\binom{2}{j}\beta^j\alpha^{\frac{j}{\alpha\theta}-1}(-\mu)^{2-j}\left[\frac{\Gamma\frac{2\theta+j}{\alpha\theta},\Gamma\frac{\alpha\theta+j}{\alpha\theta}}{\Gamma\frac{2\theta+2j+\alpha\theta}{\alpha\theta}}\right]\end{array}\right\}}}{\left(\frac{2\beta^2}{\theta}\alpha^{\frac{1-2\alpha\theta}{\alpha\theta}}\left[\frac{\Gamma\frac{2\theta+1}{\alpha\theta}\Gamma\frac{\alpha\theta-1}{\alpha\theta}}{\Gamma\frac{2+\alpha}{\alpha}}\right]\right)(1+\gamma)+\gamma\beta^1\alpha^{\frac{1}{\alpha\theta}-1}\left[\frac{\Gamma\frac{1+\theta}{\alpha\theta}\Gamma 1-\frac{1}{\alpha\theta}}{\Gamma\frac{\alpha+1}{\alpha}}\right]} \quad (17)$$

3.5. Coefficient of Skewness

The coefficient of skewness provides valuable information about the shape of a dataset's distribution and is widely used in various fields, including finance, economics, and social sciences. Mathematically, the formula for calculating the coefficient of skewness (SK) is:

$$(SK)_{TSK} = \frac{\mu_3}{(\mu_2)^{\frac{3}{2}}}$$

$$(SK)_{TSK} = \frac{\left\{\begin{array}{l} \beta^j \alpha^{\frac{j}{\alpha\theta}-1} \sum_{j=0}^{3} \binom{3}{j} (-\mu)^{3-j} \beta^j \alpha^{\frac{j}{\alpha\theta}-1} \left[\frac{\Gamma \frac{j}{\alpha\theta} + \frac{1}{\alpha} \Gamma 1 - \frac{j}{\alpha\theta}}{\Gamma 1 + \frac{1}{\alpha}}\right] (2+\gamma) \\ - \frac{2\alpha\theta(1+\gamma)}{\beta} \frac{\beta^4}{\alpha\theta} \alpha^{\frac{r-\theta}{\alpha\theta}} \sum_{j=0}^{3} \binom{3}{j} \beta^j \alpha^{\frac{j}{\alpha\theta}-1} (-\mu)^{3-j} \left[\frac{\Gamma \frac{2\theta+j}{\alpha\theta}, \Gamma \frac{\alpha\theta+j}{\alpha\theta}}{\Gamma \frac{2\theta+2j+\alpha\theta}{\alpha\theta}}\right] \end{array}\right\}}{\left(\left\{\begin{array}{l} \beta^j \alpha^{\frac{j}{\alpha\theta}-1} \sum_{j=0}^{2} \binom{2}{j} (-\mu)^{2-j} \beta^j \alpha^{\frac{j}{\alpha\theta}-1} \left[\frac{\Gamma \frac{j}{\alpha\theta} + \frac{1}{\alpha} \Gamma 1 - \frac{j}{\alpha\theta}}{\Gamma 1 + \frac{1}{\alpha}}\right] (2+\gamma) \\ - \frac{2\alpha\theta(1+\gamma)}{\beta} \frac{\beta^3}{\alpha\theta} \alpha^{\frac{2-\theta}{\alpha\theta}} \sum_{j=0}^{2} \binom{2}{j} \beta^j \alpha^{\frac{j}{\alpha\theta}-1} (-\mu)^{2-j} \left[\frac{\Gamma \frac{2\theta+j}{\alpha\theta}, \Gamma \frac{\alpha\theta+j}{\alpha\theta}}{\Gamma \frac{2\theta+2j+\alpha\theta}{\alpha\theta}}\right] \end{array}\right\}\right)^{\frac{3}{2}}} \quad (18)$$

3.6. Coefficient of Kurtosis

The coefficient of kurtosis helps to understand the shapes of the distribution and can be useful in various fields like statistics, risk analysis, and finance. Mathematically, the formula for calculating the coefficient of kurtosis (KU) is:

$$(CK)_{TSK} = \frac{E(X-\mu)^4}{\sigma^4}$$

$$(\mathrm{CK})_{\mathrm{TSK}}=\frac{\left\{\begin{array}{l}\beta^{j}\alpha^{\frac{j}{\alpha\theta}-1}\sum_{j=0}^{4}\binom{4}{j}(-\mu)^{4-j}\beta^{j}\alpha^{\frac{j}{\alpha\theta}-1}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{1}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}}{\Gamma 1+\frac{1}{\alpha}}\right](2+\gamma)\\-\frac{2\alpha\theta(1+\gamma)}{\beta}\frac{\beta^{5}}{\alpha\theta}\alpha^{\frac{4-\theta}{\alpha\theta}}\sum_{j=0}^{4}\binom{4}{j}\beta^{j}\alpha^{\frac{j}{\alpha\theta}-1}(-\mu)^{4-j}\left[\frac{\Gamma\frac{2\theta+j}{\alpha\theta},\Gamma\frac{\alpha\theta+j}{\alpha\theta}}{\Gamma\frac{2\theta+2j+\alpha\theta}{\alpha\theta}}\right]\end{array}\right\}}{\left[\left\{\begin{array}{l}\beta^{j}\alpha^{\frac{j}{\alpha\theta}-1}\sum_{j=0}^{2}\binom{2}{j}(-\mu)^{2-j}\beta^{j}\alpha^{\frac{j}{\alpha\theta}-1}\left[\frac{\Gamma\frac{j}{\alpha\theta}+\frac{1}{\alpha}\Gamma 1-\frac{j}{\alpha\theta}}{\Gamma 1+\frac{1}{\alpha}}\right](2+\gamma)\\-\frac{2\alpha\theta(1+\gamma)}{\beta}\frac{\beta^{3}}{\alpha\theta}\alpha^{\frac{2-\theta}{\alpha\theta}}\sum_{j=0}^{2}\binom{2}{j}\beta^{j}\alpha^{\frac{j}{\alpha\theta}-1}(-\mu)^{2-j}\left[\frac{\Gamma\frac{2\theta+j}{\alpha\theta},\Gamma\frac{\alpha\theta+j}{\alpha\theta}}{\Gamma\frac{2\theta+2j+\alpha\theta}{\alpha\theta}}\right]\end{array}\right\}\right]^{2}} \quad (19)$$

4. MOMENT GENERATING FUNCTION

Given a random variable X~ TSK, the moment generating function (mgf) is defined as:

$$M_X(t)=E(\mathrm{e}^{tX})=\int_0^\infty \mathrm{e}^{tx}f(x)\,dx \qquad (20)$$

Which can be written as

$$M_X(t)=\sum_{r=0}^{\infty}\frac{t^r}{r!}\mu'_r$$

So, we obtain

$$M_X(t)=\sum_{r=0}^{\infty}\frac{t^r}{r!}\left(\frac{2\beta^{r+1}}{\theta}\alpha^{\frac{r-2\theta\alpha}{\theta\alpha}}\left[\frac{\Gamma\frac{2\theta+r}{\theta\alpha}\Gamma\frac{\theta\alpha-r}{\theta\alpha}}{\Gamma\frac{2+\alpha}{\alpha}}\right]\right)(1+\gamma)$$
$$+\gamma\beta^{r}\alpha^{\frac{r}{\alpha\theta}-1}\left[\frac{\Gamma\frac{r+\theta}{\alpha\theta}\Gamma 1-\frac{r}{\alpha\theta}}{\Gamma\frac{\alpha+1}{\alpha}}\right] \quad (21)$$

The characteristic function of TSK, can be obtained as:

$$M_X(it)=\sum_{r=0}^{\infty}\frac{ti^r}{r!}\left(\frac{2\beta^{r+1}}{\theta}\alpha^{\frac{r-2\theta\alpha}{\theta\alpha}}\left[\frac{\Gamma\frac{2\theta+r}{\theta\alpha}\Gamma\frac{\theta\alpha-r}{\theta\alpha}}{\Gamma\frac{2+\alpha}{\alpha}}\right]\right)(1+\gamma)+\gamma\beta^r\alpha^{\frac{r}{\alpha\theta}-1}\left[\frac{\Gamma\frac{r+\theta}{\alpha\theta}\Gamma 1-\frac{r}{\alpha\theta}}{\Gamma\frac{\alpha+1}{\alpha}}\right] \qquad (22)$$

5. PARAMETER ESTIMATION

Let $X_1, X_2, \dots, X_n$ be a random sample of size n from TSK. Then the likelihood function from [11] can be written as:

$$L(\boldsymbol{x};\alpha,\theta,\beta,\gamma)=\prod_{i=1}^{n} f(x_i;\alpha,\theta,\beta,\gamma) \qquad (23)$$

$$=\prod_{i=1}^{n}\left(\frac{\alpha\theta}{\beta}\left(\frac{x_i}{\beta}\right)^{\theta\text{-}1}\left[\alpha+\left(\frac{x_i}{\beta}\right)^{\alpha\theta}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\left[2(1+\gamma)\left(1-\left[\frac{\left(\frac{x_i}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{x_i}{\beta}\right)^{\alpha\theta}}\right]^{\left(\frac{1}{\alpha}\right)}\right)-\gamma\right]\right) \qquad (24)$$

Accumulation taking logarithm of Eq.(24), and the log-likelihood can be written as:

$$\ell(\beta,\alpha,\theta,\gamma)=\ln L(x,\beta,\alpha,\theta,\gamma)\ =\ln\prod_{i=1}^{n}\left(\frac{\alpha\theta}{\beta}\left(\frac{x_i}{\beta}\right)^{\theta\text{-}1}\left[\alpha+\left(\frac{x_i}{\beta}\right)^{\alpha\theta}\right]^{-\left(\frac{\alpha+1}{\alpha}\right)}\left[2(1+\gamma)\left(1-\left[\frac{\left(\frac{x_i}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{x_i}{\beta}\right)^{\alpha\theta}}\right]^{\left(\frac{1}{\alpha}\right)}\right)-\gamma\right]\right)$$

$$=\sum_{i=1}^{n}\left\{\ln\alpha+\ln\theta\ \text{-}\theta\ln\beta+(\theta\text{-}1)\ln x_i-\left(\frac{\alpha+1}{\alpha}\right)\ln\left[\alpha+\left(\frac{x_i}{\beta}\right)^{\alpha\theta}\right]\right.$$
$$\left.+\ln\left[2(1+\gamma)\left(1-\left[\frac{\left(\frac{x_i}{\beta}\right)^{\alpha\theta}}{\alpha+\left(\frac{x_i}{\beta}\right)^{\alpha\theta}}\right]^{\left(\frac{1}{\alpha}\right)}\right)-\gamma\right]\right\} \qquad (25)$$

By taking the first partial derivatives of the log-likelihood function with respect to the four parameters $(\beta, \alpha, \theta, \gamma)$ as follows:

$$\frac{\partial \ell(\beta,\alpha,\theta,\gamma)}{\partial \alpha} = \sum_{i=1}^{n} \left(\frac{1}{\alpha} + \frac{\ln[\alpha + (\frac{x_i}{\beta})^{\alpha\theta}]}{\alpha^2} - \frac{(1+\alpha)(1+\left(\frac{x_i}{\beta}\right)^{\alpha\theta} \theta \ln[\frac{x}{\beta}])}{\alpha(\alpha + (\frac{x_i}{\beta})^{\alpha\theta})} - \frac{2\left(\frac{\alpha}{\alpha+\left(\frac{x_i}{\beta}\right)^{\alpha\theta}}\right)^{1+\frac{1}{\alpha}} (1+\gamma)\left(\left(\alpha + \left(\frac{x_i}{\beta}\right)^{\alpha\theta}\right) \ln\left[\frac{\alpha}{\alpha+\left(\frac{x_i}{\beta}\right)^{\alpha\theta}}\right] + \left(\frac{x_i}{\beta}\right)^{\alpha\theta} (-1 + \alpha\theta \ln[\frac{x_i}{\beta}])\right)}{\alpha^3 (2(\frac{\alpha}{\alpha+(\frac{x_i}{\beta})^{\alpha\theta}})^{\frac{1}{\alpha}} + (-1 + 2(\frac{\alpha}{\alpha+(\frac{x_i}{\beta})^{\alpha\theta}})^{\frac{1}{\alpha}})\gamma)} \right) \quad (26)$$

$$\frac{\partial \ell(\beta,\alpha,\theta,\gamma)}{\partial \theta} = \sum_{i=1}^{n} \left(\frac{1}{\theta} - \frac{\alpha\left(-1 + \left(\frac{x_i}{\beta}\right)^{\alpha\theta}\right) \ln[\frac{x_i}{\beta}]}{\alpha + (\frac{x_i}{\beta})^{\alpha\theta}} - \frac{2(\frac{\alpha}{\alpha+(\frac{x_i}{\beta})^{\alpha\theta}})^{1+\frac{1}{\alpha}} (\frac{x_i}{\beta})^{\alpha\theta} (1+\gamma) \ln[\frac{x_i}{\beta}]}{\alpha(2(\frac{\alpha}{\alpha+(\frac{x_i}{\beta})^{\alpha\theta}})^{\frac{1}{\alpha}} + (-1 + 2(\frac{\alpha}{\alpha+(\frac{x_i}{\beta})^{\alpha\theta}})^{\frac{1}{\alpha}})\gamma)} \right) \quad (27)$$

$$\frac{\partial \ell(\beta,\alpha,\theta,\gamma)}{\partial \theta} = \sum_{i=1}^{n} \left(\frac{\theta\left(\alpha(-1 + (\frac{x_i}{\beta})^{\alpha\theta}) + \frac{2(\frac{\alpha}{\alpha+(\frac{x_i}{\beta})^{\alpha\theta}})^{\frac{1}{\alpha}} (\frac{x_i}{\beta})^{\alpha\theta} (1+\gamma)}{2(\frac{\alpha}{\alpha+(\frac{x_i}{\beta})^{\alpha\theta}})^{\frac{1}{\alpha}} + (-1 + 2(\frac{\alpha}{\alpha+(\frac{x_i}{\beta})^{\alpha\theta}})^{\frac{1}{\alpha}})\gamma}\right)}{(\alpha + (\frac{x_i}{\beta})^{\alpha\theta})\beta} \right) \quad (28)$$

$$\frac{\partial \ell(\beta,\alpha,\theta,\gamma)}{\partial \gamma} = \sum_{i=1}^{n} \left(\frac{(-1 + 2(\frac{\alpha}{\alpha+(\frac{x_i}{\beta})^{\alpha\theta}})^{\frac{1}{\alpha}})}{2(\frac{\alpha}{\alpha+(\frac{x_i}{\beta})^{\alpha\theta}})^{\frac{1}{\alpha}} - \gamma + 2(\frac{\alpha}{\alpha+(\frac{x_i}{\beta})^{\alpha\theta}})^{\frac{1}{\alpha}}\gamma} \right) \quad (29)$$

The estimator's $(\hat{\beta}, \hat{\gamma}, \hat{\theta}, \hat{\alpha})$ can be obtained by solving this system numerically after making the equations equal to zero, i.e. $\frac{\partial \ell}{\partial \alpha} = 0, \frac{\partial \ell}{\partial \theta} = 0, \frac{\partial \ell}{\partial \beta} = 0, \frac{\partial \ell}{\partial \gamma} = 0.$

6.APPLICATION OF TRANSMUTED SURVIVAL KAPPA DISTRIBUTION

The TSK is characterized by flexibility compared to several other models, such as the one-parameter exponential distribution (ED), three-parameter Lindley distribution (TPLD), three-parameter Kappa distribution (KD), and Weibull distribution (WD). The analysis involved fitting these distributions to a dataset representing the number of weeks that patients with heart disease stayed in the hospital before their demise at (Al-Hussein Educational Hospital in Karbala), with n=104 see (Table 1).

By using metrics Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Akaike Information Criterion Corrected (AICC) for the new distribution performance was compared with Kappa Distribution, Lindely three-parametric, Weibull distribution, and Weibull Pareto distribution. The new distribution obtained the lowest AIC and AICC values and was considered the most flexible and superior for the given dataset. see (Table 2). The goodness of fit of the proposed model tells how well it fits with real observations. See Figure (4).

Table 1. Dataset for the number of hours patients were in hospital before death

0.1	0.3	1.2	1.4	1.6	2	2.6	3	3.5	4.1	4.8
0.1	0.3	1.2	1.4	1.7	2.1	2.7	3.1	3.6	4.3	4.8
0.1	0.4	1.2	1.5	1.7	2.2	2.7	3.1	3.6	4.4	4.9
0.2	0.4	1.3	1.5	1.7	2.4	2.8	3.1	3.6	4.4	4.9
0.2	0.4	1.3	1.5	1.7	2.4	2.8	3.1	3.7	4.4	
0.2	0.4	1.3	1.5	1.8	2.4	2.9	3.2	3.8	4.5	
0.2	0.4	1.3	1.5	1.8	2.5	2.9	3.2	3.9	4.5	
0.3	1	1.3	1.6	1.9	2.6	2.9	3.2	4	4.6	
0.3	1	1.4	1.6	2	2.6	2.9	3.3	4	4.6	
0.3	1.2	1.4	1.6	2	2.6	3	3.4	4	4.7	

From Table (2) we found the best model conforming to the lowest value for Akaike Information Criterion (AIC), Akaike Information Correct (AICc) and Bayesian Information Criterion (BIC) is the new distribution compared others distributions to use in this paper.

Table 2. MLE Estimates and Criterion Values X^2 Anderson-D, Cramer-V and AIC , AIC_C, BIC.

Distributions	MLE	X^2 Anderson-D		Cramer- V		AIC	AIC_C	BIC
		Statistic	P-Value	statistic	P-Value			
TSK	$\hat{\alpha} = 0.46421$ $\hat{\gamma} = 3.528$ $\hat{\theta} = 2.64572$ $\hat{\beta} = 5.786$	3.184	0.0527	3.163	0.33911	301.77	301.14	308.524
KD	$\hat{\hat{\alpha}} = 0.421$ $\hat{\beta} = 0.9761$ $\hat{\theta} = 2.3467$	6.613	0.0665	9.159	0.7391	457.72	457.79	465.321
LTD	$\hat{\alpha} = 2.169$ $\hat{\beta} = 1.3467$ $\hat{\theta} = 1.781$	8.037	0.0867	3.0565	0.091674	402.10	402.18	406.424
ED	$\hat{\gamma} = 0.0918$	4.561	0.0929	6.23	0.0517	503.14	503.18	505.340
WD	$\hat{\alpha} = 2.031$ $\hat{\theta} = 0.0421$	2.235	0.0926	0.533	0.91549	446.88	446.94	452.094

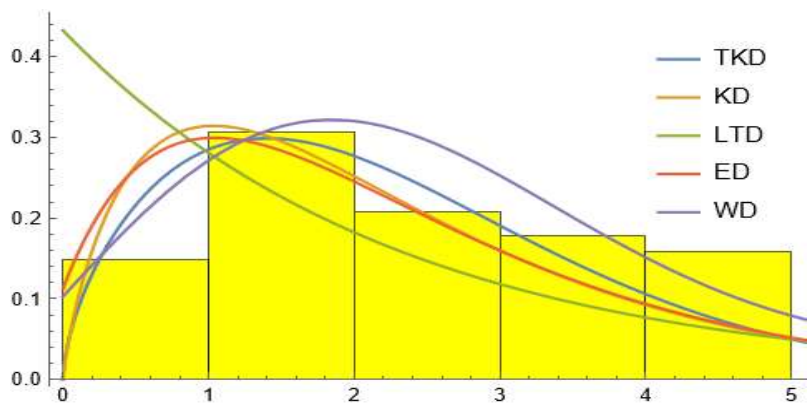

Figure 4. fitted pdf's on histogram.

7. CONCLUSION

In this paper, a novel probability distribution is introduced. The new distribution is a Transmuted Survival Kappa (TSK) distribution. The characteristics of new distribution are found that TSK distribution gives a better fit to the dataset as compared with exponential, Kappa Distribution, Lindely three parametric, Weibull distribution, and Weibull Pareto distribution further. The TSK distribution can be applied to various areas.

The TSK distribution and distribution may be suitable for most of the lifetime data and provide better outcomes than other distributions.

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